Extending the Johnson-Neyman Procedure to Categorical Independent Variables: Mathematical Derivations and Computational Tools

A Thesis

Presented in Partial Fulfillment of the Requirements for the Degree Master of Arts in the Graduate School of The Ohio State University

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2016

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Abstract

Moderation analysis is used throughout many scientific fields, including psychology and other social sciences, to model contingencies in the relationship between some independent variable (X) and some outcome variable (Y) as a function of some other variable, typically called a moderator (M). Inferential methods for testing moderation provide only a simple yes/no decision about whether the relationship is contingent. These contingencies can often be complicated. Researcher often need to look closer. Probing the relationship between X and Y at different values of the moderator provides the researcher with a better understanding of how the relationship changes across the moderator. There are two popular methods for probing an interaction: simple slopes analysis and the Johnson-Neyman procedure. The Johnson-Neyman procedure is used to identify the point(s) along a continuous moderator where the relationship between the independent variable and the outcome variable transition(s) between being statistically significant to nonsignificant or vice versa. Implementation of the Johnson-Neyman procedure when X is either dichotomous of continuous is well described in the literature; however, when X is a multicategorical variable it is not clear how to implement this method. I begin with a review of moderation and popular probing techniques for dichotomous and continuous X. Next, I derive the Johnson-Neyman solutions for three groups and continue with a partial derivation for four groups. Solutions for the four-group derivation rely on finding the roots of

an eighth-degree polynomial for which there is no algebraic solution. I provide an iterative computer program for SPSS and SAS that solves for the Johnson-Neyman boundaries for any number of groups. I describe the performance of this program, relative to known solutions, and typical run-times under a variety of circumstances. Using a real dataset, I show how to analyze data using the tool and how to interpret the results. I conclude with some consideration about when to use and when not to use this tool, future directions, and general conclusions.

This thesis is dedicated to my friends and family who have supported me in all my various dreams. Particularly to my mother Lori for always seeing the best in me and my best friend Colleen for being a rock and support through the toughest times in life. Even though we are far apart you are always close to my heart.

Acknowledgments

I would like to thank my committee members for their detailed feedback and questions, particularly my advisor Dr. Hayes for suggesting this topic and guiding me through the research process. I would like to thank the attendees of the Quantitative Psychology Graduate Student Meeting for their helpful feedback, particularly in the early stages of development. Special thanks to J. R. R. Tolkien for inspiring the name of the tool.

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Chapter 1: Introduction

When a researcher believes that the relationship between an independent variable (X) and an outcome (Y) may depend on some other variable (M) they can test this hypotheses by allowing for moderation of the effect of X on Y by M in a regression analysis. For example, Kim and Baek (2014) were interested in if people's selective self-presentation online (X) predicted their online life satisfaction (Y), and if this relationship depended on self-esteem (M). Indeed, they found that selective self-presentation online predicted increased online life satisfaction, and this relationship was larger among those with low self-esteem than those with high self-esteem. Siy and Cheryan (2013) studied how Asian Americans reacted to positive stereotypes based on their Asian culture. They found that those who had an independent self-construal (M) as compared to an interdependent self-construal, reactived more negatively (Y) when they were positively stereotyped (X). In this study self-construal, the moderator, was measured on a single scale which ranged from interdependent to independent.

Research often begins with a simple correlation question: "Does this relate to that?". As a research area develops, these questions may gain some nuance, such as whether or not two things are *always* related in the same way, or does the relationship depend on other variables. Questions about contingencies help define boundary

conditions for the relationships between variables. These types of analysis can provide explanations for seemingly contradictory results. For example, Campbell (1990) found that boys had more positive attitudes towards computers than girls. However, DeRemer (1989) found that girls had more positive attitudes toward computers than boys. One major difference between these two studies is the age of students sampled. Campbell (1990) looked at high school students and DeRemer (1989) examined students in grades three and six. A single study which sampled students from a variety of grades could show that the relationship between gender and computer attitudes varies with age or school grade, as was shown in a meta-analysis by Whitley Jr. (1997).

Researchers throughout psychology are often interested in moderators such as situational variables, individual differences, and experimental conditions. Using moderation analysis allows researchers to more clearly understand under what conditions certain effects occur or do not occur, how their magnitude varies, and how their direction can change. Moderation analyses are important not only for improving theory, but also for improving practical applications. For example, given the results of a moderation analysis, it has been suggested that practitioners can assign individuals to treatments, such as educational classes, where they are predicted to have the most beneficial outcomes given their scores on the moderator (Forster, 1971).

Statistical moderation analysis has been used in psychology for many years and is taught in introductory regression classes to most graduate students in the field. Many books have been written on the topic of moderation and interactions (e.g., Jaccard & Turrisi, 2003; Aiken & West, 1991) and complete chapters and full sections of introductory regression and statistics books are dedicated to this topic (e.g., Cohen, Cohen, West, & Aiken, 2003; Field, 2013; Darlington & Hayes, 2017; Hayes, 2013;

Howell, 2007). Researchers also often use statistical methods to probe interactions to better understand the nature of the contingent effect they are interested in. By expanding methods for probing interactions this thesis provides additional tools for psychology researchers to answer the questions they are interested in.

The aim of this thesis is to provide a tool to help researchers probe moderated relationships when the independent variable of interest is categorical, particularly having three or more categories, using the Johnson-Neyman procedure. Moderation analysis involves both inferential methods for decisions of moderation or no moderation and probing methods for investigating the nature of the moderated relationship. The topic of moderation with categorical independent variables has been discussed in a variety of books and publications (e.g., Cohen et al., 2003; Huitema, 1980; Spiller, Fitzimons, Lynch Jr., & McClelland, 2013); however, probing methods for categorical predictors are less frequently discussed. Common probing methods for examining moderated relationships include two approaches: the simple slopes approach and the Johnson-Neyman procedure. Some researchers have discussed how to apply the simple slopes approach for moderated relationships with categorical independent variables and either dichotomous or continuous moderators (Hayes, 2013; Spiller et al., 2013). The primary contribution of this thesis is to describe how the the Johnson-Neyman procedure can be generalized to situations where the independent variable is categorical and the moderator is continuous. This case is particularly of interest to researchers focused on the effect of the independent variable on the outcome at different values of the moderator, rather than an estimate of the effect of the moderator on the outcome at each level of the independent variable.

I begin with an overview of the procedures for testing moderation as well as the two methods for probing moderated relationships: simple slopes analysis and the Johnson-Neyman procedure. I will focus on a model comparison approach, using ordinary least squares (OLS) for model estimation. I will then give a brief discussion of the historical development of the Johnson-Neyman procedure. I continue by describing how to estimate competing models in order to test moderation hypotheses with categorical independent variables and implementation of the simple slopes method in these cases. I then provide an analytical derivation of the Johnson-Neyman procedure with a 3-group categorical variable and then start the derivation for a 4-group categorical variable. As the number of groups (k) increases, the Johnson-Neyman procedure relies on finding the roots of a 2^{k-1} degree polynomial. By the Abel-Ruffini theorem, there are no general algebraic solutions for polynomial equations of degree five or higher (Abel, 1824; Ruffini, 1799), meaning Johnson-Neyman solutions do not have closed forms and cannot be found using traditional methods. I propose an iterative computational method which finds the Johnson-Neyman solutions, and will provide a tool, OGRS, to make this analysis easy to do using SPSS or SAS. I will illustrate how to use this tool and interpret the results using two examples of real data from psychology.

Chapter 2: Linear Moderation Analysis in OLS Regression

Moderation analysis using ordinary least squares regression (OLS) has two major parts. First, a researchers tests if there is sufficient evidence that the relationship between the independent variable and the outcome depends on some moderator. If evidence of moderation in found, researchers often *probe* the interaction in order to better understand and visualize the contingent relationship. This practice is much like the practice of examining simple effects in ANOVA. In this chapter I describe common methods for both inference about moderation and probing interactions in linear moderation analysis using OLS regression.

2.1 Inference about Moderation

Though moderation can be tested in a variety of ways, the focus of this paper will be using OLS regression for model estimation. Linear moderation is traditionally tested by estimating and comparing the fit of two models, one with no contingent relationships and one which allows for contingent relationships.

Model 1: $Y_i = b_0^* + b_1^* X_i + b_2^* M_i + \epsilon_i^*$ where $\epsilon_i \overset{iid}{\sim} N(0, \sigma^{*2})$

Model 2: $Y_i = b_0 + \Theta_{X \to Y|M} X_i + b_2 M_i + \epsilon_i$ where $\epsilon_i \overset{iid}{\sim} N(0, \sigma^2)$

$$\Theta_{X \to Y|M} = b_1 + b_3 M_i$$

In Models 1 and 2, Y is a continuous outcome variable which is being predicted. The predictor variables in the models are X and M's. Within the context of moderation, it is often helpful to frame the problem with respect to an independent variable, the variable whose effect on Y is of interest, and a moderator, the variable which is believed to influence the independent variable's effect on Y. I will use X as the independent variable and M as the moderator variable throughout. In Model 1, b_0^*, b_1^*, and b_2^*, are the population regression coefficients. In Model 2, b_0, b_1, b_2, and b_3 are the population regression coefficients. The stars in Model 1 are meant to indicate that the coefficients are different from the coefficients in Model 2. In Model 1 the effect of X on Y is not contingent on M but, rather, is constant, b_1^*. In Model 2 however, the effect of X on Y is $\Theta_{X \rightarrow Y|M}$, the conditional effect of X on Y at some value of M, which is defined as a linear function of M. In this way, X's effect on Y depends on M, and is allowing for a certain type of moderation, *linear* moderation. The effect $\Theta_{X \rightarrow Y|M}$ could be any defined function of M, such as a quadratic function, but because linear moderation is most common in psychology, this thesis will focus solely on linear moderation. Also note that the error terms in these equations are assumed to be normally distributed with a constant variance σ^2 or σ^{*2}. This will be the case throughout this thesis but to avoid needless repetition this notation will be omitted in future equations.

Model 1 is nested within Model 2. Specifically, if the b_3 parameter is zero in the population, then Model 2 simplifies to Model 1. This can be shown by plugging $\Theta_{X \rightarrow Y|M}$ into the equation for Model 2 and expanding terms:

$$Y_i = b_0 + b_1 X_i + b_2 M_i + b_3 X_i M_i + \epsilon_i$$

By setting $b_3 = 0$:

$$Y_i = b_0 + b_1 X_i + b_2 M_i + 0 X_i M_i + \epsilon_i$$

$$Y_i = b_0 + b_1 X_i + b_2 M_i + \epsilon_i$$

This last model is the equivalent to Model 1. This shows how Model 1 is nested within Model 2.

The most common way to test for linear moderation in this case is to test if b_3 is significantly different from zero in Model 2. If this coefficient is not significantly different from zero, there is insufficient evidence that allowing the effect of X on Y to depend on M improves the fit of the model, and so it is more reasonable to say this relationship is not contingent.

An equivalent way to test for linear moderation is to use hierarchical OLS regression. Though this method may seem excessively complicated as compared to testing one coefficient, this method generalizes to the case of a categorical X whereas the test of b_3 does not. Because of this, I will focus on hierarchical OLS as the method of inference for moderation effects. A researcher would first estimate Model 1 then add the product term, $X_i M_i$, to estimate Model 2. Comparing these two models will test if allowing the relationship between X and Y to be contingent on M explains additional variance in the outcome variable. This type of analysis can be completed using any number of statistical packages. An F statistic corresponding to the change in the variance explained can be calculated using Equation 2.1.

$$F = \frac{df_2(R_2^2 - R_1^2)}{q(1 - R_2^2)} \tag{2.1}$$

Here the subscript on the R^2 and df refer to the model number: Models 1 and 2 respectively as described above. The residual degrees of freedom in Model 2 is

noted by df_2. The variance explained in Y in Model 1 and Model 2, are R_1^2 and R_2^2 respectively, and q is the number of constraints made to Model 2 to get Model 1. In the situation where X is dichotomous or continuous, q is always 1, but as we move into the case of a categorical X, q will depend on the number of categories in X. This F-value is then compared to a critical F to decide if it is significant or the cumulative distribution function is used to calculate the area to the right of the observed F-value to calculate a p-value, the probability of observing this change in R^2 assuming that the relationship between X and Y is linearly independent of M (i.e. not contingent).

An inference about whether the relationship between X and Y is dependent on M is important, but this inference does not completely describe the nature of the contingency. The relationship between X and Y may get stronger or weaker as M increases, and in order to understand the full nature of the contingency, it is important to interpret the sign and magnitude of the regression coefficients. In multiple regression without interactions, the regression coefficients are an estimate of the effect of each variable *controlling* for the other variables or holding the other variables constant. For example, in Model 1 an estimate of b_1^* would be interpreted as the expected change in Y for a one unit change in X, holding M constant.

In regression models with interactions, the interpretations of the coefficients are no longer estimates of effects *controlling* for the other variables, but rather they are conditional effects. The estimate of the coefficient b_0 has the same interpretation as b_0^*: the expected value of Y when both X and M are zero. The other coefficients, however, do not correspond with their counterparts in Model 1. They cannot be interpreted as holding the other variables constant, because a one unit change in X would also result in a change in XM when M is nonzero. In the model with interactions, b_1

can be interpreted as the expected change in Y with a one unit change in X when M is zero. Similarly, b_2 can be interpreted as the expected change in Y with a one unit change in M when X is zero. These two effects, b_1 and b_2, are conditioned on certain variables being zero. The b_3 parameter can be best understood by examining the equation for $\Theta_{X \to Y|M}$. From this equation it is clear that and estimate of b_3 is the expected change in the effect of X on Y with a one unit change in M. Therefore if b_3 is positive, the relationship between X and Y will become more positive as M increases. If b_3 is negative, the relationship between X and Y will become more negative as M increases. The magnitude of b_3 indicates how much the relationship changes with a one unit change in M.

A Note on Symmetry. Throughout this proposal I call X the "independent variable" and M the "moderator." However, these distinctions are mathematically arbitrary and driven primarily by theoretical considerations of the researcher. Alternatively, Model 2 could be used to describe how M's effect on Y may be linearly depend on X, a property called *symmetry*. Equation 2.2 results from plugging $\Theta_{X \to Y|M}$ into the equation for Model 2:

$$Y_i = b_0 + (b_1 + b_3 M_i)X_i + b_2 M_i + \epsilon_i \tag{2.2}$$

Note that by multiplying out the terms, Equation 2.2 is equivalent to Equation 2.3.

$$Y_i = b_0 + b_1 X_i + b_3 M_i X_i + b_2 M_i + \epsilon_i \tag{2.3}$$

And by regrouping the terms in a new way, it is clear that the same equation could be used to describe M's effect on Y as a function of X.

$$Y_i = b_0 + b_1 X_i + (b_2 + b_3 X_i)M_i + \epsilon_i \tag{2.4}$$

Where the conditional effect of M on Y could be described as $\Theta_{M \to Y|X} = b_2 + b_3 X_i$. There is no mathematical distinction between X's effect being moderated by M and M's effect being moderated by X. So, throughout the proposal I will refer to X as the independent variable and M as the moderator with the understanding that this distinction is for simplicity and depending on the research question, researchers should consider which assignment of independent variable and moderator would be more useful and informative to their research question.

2.2 Probing Moderation Effects

Once an inferential test of moderation is completed and evidence of moderation is found, researchers often ask more specific questions about the nature of the moderated effect. For what values of M does X positively influence Y, and for what values of M does X negatively influence Y? When M is at its mean, does X significantly predict Y? These are all questions related to conditional effects, the effect of X on Y conditional on some value of M. Questions of this nature can be answered by "probing" interactions. Throughout this manuscript I will discuss two frequently used methods for probing an interaction: simple slopes analysis and the Johnson-Neyman procedure.

2.2.1 Simple-Slopes Analysis

Simple-slopes analysis is a method for estimating and testing conditional effects in order to answer the question: When M is equal to some value, say m, what is the effect of X on Y? Simple slopes analysis relies on the estimate of the conditional effect of X on Y, $\hat{\Theta}_{X \to Y|M=m}$, and its standard error, $s_{\hat{\Theta}_{X \to Y|M=m}}$. In simple-slopes analysis, the researcher chooses a value of M to assess the effect on X on Y at m.

The selected value of M is entered into the Equations 2.5 and 2.6 to estimate the conditional effect of X on Y at m and the estimated standard error of this effect.

$$\hat{\Theta}_{X \to Y|M=m} = \hat{b}_1 + \hat{b}_3 m \tag{2.5}$$

$$\hat{s}_{\hat{\Theta}_{X \to Y|M=m}} = \sqrt{\hat{s}^2_{\hat{b}_1} + 2m\hat{s}_{\hat{b}_1\hat{b}_3} + m^2\hat{s}^2_{\hat{b}_3}} \tag{2.6}$$

The regression coefficient estimates from Model 2 are used as $\hat{b}_1$ and $\hat{b}_3$ in Equation 2.5. Estimates from Model 2 are also used in Equation 2.6: $\hat{s}^2_{\hat{b}_1}$ is the estimated sampling variance of $\hat{b}_1$, $\hat{s}^2_{\hat{b}_3}$ is the estimated sampling variance of $\hat{b}_3$, and $s_{\hat{b}_1\hat{b}_3}$ is the estimated sampling covariance between $\hat{b}_1$ and $\hat{b}_3$. The ratio of $\hat{\Theta}_{X \to Y|M=m}$ to $\hat{s}_{\hat{\Theta}_{X \to Y|M=m}}$ is t-distributed with $n - p - 1$ degrees of freedom under the null hypothesis that $\hat{\Theta}_{X \to Y|M=m} = 0$. That is

$$t_{obs} = \frac{\hat{\Theta}_{X \to Y|M=m}}{\hat{s}_{\hat{\Theta}_{X \to Y|M=m}}} \sim t_{(n-p-1)} \mid H_0$$

where n is the total sample size and p is the number of predictors in the unconstrained model. For example, in Model 2, there are three regressors (X, M, and XM), so $p = 3$.

In simple-slopes analysis, m is chosen and plugged in to Equations 2.5 and 2.6, then the observed t-value, t_{obs}, is calculated. This value is then compared to a critical t-value corresponding to the $\frac{\alpha}{2}$ quantile of the t-distribution with $n - p - 1$ degrees of freedom, where α is the level of the test corresponding to the desired Type I error rate of the test, typically chosen as .01, .05, or .1. If the observed t-value is more extreme than the critical t-value, then the researcher concludes that is it unlikely that X has no effect on Y when $M = m$. More typically the t-statistic is used to calculate

a p-value which represents the probability that a value this extreme or more extreme would have occured under the null hypothesis. This p-value can be compared to a set α level, and if it is smaller than α the null hypothesis is rejected. Using this procedure researchers can probe the effect of X on Y at different values of M, both estimating the effect of X on Y at that value and completing a hypothesis test which determines if this effect is significantly different than zero.

Choosing points along M to probe the relationship between X and Y is often arbitrary. If M is a dichotomous variable, then it makes sense to examine the effect of X on Y for each coded value of M. If M is a continuous variable, however the choice is more arbitrary. Researchers often choose the sample mean of M and the sample mean plus and minus one standard deviation (Bauer & Curran, 2005; Cohen et al., 2003; Spiller et al., 2013). In some cases, particularly if M is skewed, one of these points may be out of the range of the collected data, and therefore claims about the estimated effect of X on Y at that point on M are dubious at best. Hayes (2013) recommends probing along the percentiles of M (e.g., 10th percentile, 20th percentile, 90th percentile) to guarantee that all probed points are within the range of the observed data on the moderator. Alternatively, there may be specific points that are of interest to researchers. For example, many depression scales have cut-off scores for the diagnosis of depression, so it may be of interest for a researcher interested in the moderating role of depression to examine the effect of their independent variable on their outcome variable at that cutoff. Similarly, some scales like BMI have ranges of interest: a BMI under 18.5 indicates being underweight, between 18.5 and 25 indicates normal range, etc. Researchers interested in the moderating role of BMI

may use these ranges to inform the points at which they probe their interaction effects using the simple slopes method.

The simple-slopes method is very helpful for understanding interaction effects by examining more closely specific conditional effects. The interpretations of these analyses often depend on the choices of the analyst, specifically at which points to probe the relationship between X and Y. Next I will discuss a method for probing interactions which does not rely on choice of sometimes arbitrary points. Rather, this method identifies points along a continuous moderator where the conditional effect of X on Y transitions from statistically significant to non-significant or vice versa.

2.2.2 The Johnson-Neyman Procedure

Rather than conditioning on specific values of the moderator, the Johnson-Neyman procedure solves for values of the moderator which mark the transition between significant and non-significant effects of X on Y. These points may be of particular interest to some researchers. They are the points, m_{JN}, along M where the conditional effect of X on Y is exactly statistically significant at level α. The same definition of the conditional effect of X on Y is used in the Johnson-Neyman procedure as in the simple slopes method; however this method, rather than plugging in values of M, sets the ratio of the conditional effect to its standard error equal to a specific value then solves for M. In order for the conditional effect of X on Y at some value of M to be exactly statistically significant at level α, then the ratio of $\hat{\Theta}_{X\rightarrow Y|M=m_{JN}}$to $\hat{s}_{\hat{\Theta}_{X\rightarrow Y|M=m_{JN}}}$ must equal exactly the critical t-value for a level α test with $n-(p+1)$ degrees of freedom.

$$\frac{\hat{\Theta}_{X\rightarrow Y|M=m_{JN}}}{\hat{s}_{\hat{\Theta}_{X\rightarrow Y|M=m_{JN}}}} = \frac{\hat{b}_1 + \hat{b}_3 m_{JN}}{\sqrt{\hat{s}^2_{\hat{b}_1} + 2m_{JN}\hat{s}_{\hat{b}_1\hat{b}_3} + m^2_{JN}\hat{s}^2_{\hat{b}_3}}} = t_{crit} = t_{n-(p+1),\alpha/2}$$

The above equation can be rewritten as a second degree polynomial.

$$0 = \hat{b}^2_1 - t^2_{crit}\hat{s}^2_{\hat{b}_1} + (2\hat{b}_1\hat{b}_3 - 2t^2_{crit}\hat{s}_{\hat{b}_1\hat{b}_3})m_{JN} + (\hat{b}^2_3 - \hat{s}^2_{\hat{b}_3}t^2_{crit})m^2_{JN} \tag{2.7}$$

Solutions to the roots of this polynomial, and therefore solutions for the Johnson-Neyman procedures, can be found using the quadratic equation. Plugging in the values of the estimated regression coefficients, sampling variances, and the critical t-value identifies the points such that the conditional effect of X on Y at m_{JN} are exactly statistically significant at level α.

$$m_{JN} = \frac{2t^2_{crit}\hat{s}_{\hat{b}_1\hat{b}_3} - 2\hat{b}_1\hat{b}_3 \pm \sqrt{(2\hat{b}_1\hat{b}_3 - 2t^2_{crit}\hat{s}_{\hat{b}_1\hat{b}_3})^2 - 4(\hat{b}^2_1 - t^2_{crit}\hat{s}^2_{\hat{b}_1})(\hat{b}^2_3 - \hat{s}^2_{\hat{b}_3}t^2_{crit})}}{2(\hat{b}^2_1 - t^2_{crit}\hat{s}^2_{\hat{b}_1})} \tag{2.8}$$

The above equation results in two Johnson-Neyman solutions, one corresponding to when the ratio of the conditional effect of X on Y to its standard error is equal to t_{crit} and one for when the ratio is equal to $-t_{crit}$. These points may or may not be within the measured range on M, and should only be interpreted if they are within the measured range of M.

Though I've described the Johnson-Neyman procedure within the context of linear regression, this is not how the original method was developed. Over time the Johnson-Neyman procedure has been generalized to more moderators and to linear regression and the general linear model. In the next section I will describe these developments and how they may be used in the creation of a method for probing interactions

between a categorical variable with three or more levels and a continuous moderator, providing an omnibus test of group differences at different values of a moderator.

A Brief History of the Johnson-Neyman Procedure

The Johnson-Neyman procedure was developed within the framework of analysis of covariance (Johnson & Neyman, 1936; Johnson & Fay, 1950). The original approach was developed in a two-group two-moderator model. They began by defining a linear model of the outcome variable of interest for two groups, group A and B:

$$E(Y_A) = a_0 + a_1 X_i + a_2 Z_i$$

$$E(Y_B) = b_0 + b_1 X_i + b_2 Z_i$$

Here Y_A and Y_B are the outcome variables for group A and B respectively. The variables X and Z are moderators measured for each individual/case. The lower case a's and b's are weights, estimated using a least squares criterion. The question was then posed: For what values of X and Z are the expected values of Y_A and Y_B the same and for which are they different? Johnson and Neyman derive the sums of squares (SS_{Full}) for a model where the expected values of Y_A and Y_B are different at specified values of X and Z, x and z, and the sums of squares ($SS_{reduced}$) for a model where the expected values are fixed to be equal at x and z. Note that SS_{Full} does not depend on the choice of x and z, because there are no constraints on this model. However, $SS_{reduced}$ is a function of x and z. This is equivalent to asking if there is an effect of group (A vs. B) for individuals with the specific observed values x and z. They define a sufficient statistic to test this question, the ratio between the two calculated sums of squares ($SS_{Full}/SS_{reduced}$) whose cumulative distribution

function is the incomplete beta function, under the null hypothesis of no difference in expected values at x and z. Johnson and Neyman define a critical value of the incomplete beta function, as a function of some test level α, then derive the region for which $SS_{Full}/SS_{reduced}$ will be smaller than that critical value. This region is defined as the region of significance and defines the region(s) of the range of the two moderators X and Z for which the expected values of Y_A and Y_B differ. Of particular importance in this method is the curve which limits the region of significance, which is defined by the points at which $SS_{Full}/SS_{reduced}$ exactly equals the critical value as defined by α. This curve is frequently referred to as the *boundary of significance*.

Extensions of the Johnson-Neyman procedure within the ANCOVA framework allowed for increased use of this method. The original work described only two moderators, but Johnson and Hoyt (1947) generalized this approach to three moderators. Abelson (1953) proposed that instead of always using the Johnson-Neyman procedure, researchers should test if the regression slopes are the same for each group (a test of moderation) and only proceed with the Johnson-Neyman procedure if this hypothesis is rejected. Otherwise researchers can use ANCOVA. Additionally, Abelson (1953) derived formulas for both the region of significance and the boundary of significance for any number of moderators. Potthoff (1964) proposed the first approach for dealing with more than two groups. He derived a simultaneous Johnson-Neyman solution for all pairwise comparisons of groups.

There have been a few proposed methods for how to treat multiple groups when using the Johnson-Neyman procedure. Huitema (1980) proposed using the procedure for each pair of groups, thus defining regions of significance for each pair of

groups. Potthoff (1964) derived a simultaneous method for these pairwise comparisons. However both of these approaches can result in a large number of regions, the interpretation of which can be difficult with an increasing number of groups. It could be useful for researchers to know where along the range of a moderator the groups differ from each other using an omnibus test of differences. Some research based on the general linear model (Hunka, 1995; Hunka & Leighton, 1997), provided equations for an omnibus region of significance with multiple groups in matrix form, which will be used in my derivations later. However, the closed form solutions for the region of significance were not provided, but rather, given a set of data the region of significance was solved for using Mathematica, a fairly expensive program which is not commonly used within psychology. Additionally, no more than three-groups were considered, which may be the upper limit of this method for finding omnibus groups differences.

An important extension of the Johnson-Neyman procedure moved away from just categorical independent variables and into the framework of multiple regression, which can include a categorical or continuous independent variable. Bauer and Curran (2005) were the first to derive the Johnson-Neyman procedure for a continuous by continuous variable interaction. They derived Equations 2.5 – 2.8, providing closed form equations for solutions to the Johnson-Neyman boundary of significance and thus the regions of significance. Bauer and Curran (2005) continue by deriving the approximate Johnson-Neyman boundary of significance for linear multilevel models. Preacher, Curran, and Bauer (2006) followed up with an online tool to calculate Johnson-Neyman boundaries of significance for multiple linear regression, multilevel

models, and latent curve analysis. Hayes and Matthes (2009) generalized this approach to logistic regression, where the outcome is dichotomous. These publications together allow for many applications to continuous independent variables in a variety of different types of models.

These innovations and easy to use tools have allowed for the Johnson-Neyman procedure to be applied in more varied contexts, increasing the versatility of this type of analysis. Since its conception, researchers from a variety of academic fields have published in applied journals encouraging their colleagues to consider the Johnson-Neyman procedure as an alternative to ANCOVA or the simple-slopes method for probing moderation effects, including education (Carroll & Wilson, 1970), nursing (D'Alonzo, 2004), ecology (Engqvist, 2005), psychology (Hayes & Matthes, 2009), and marketing (Spiller et al., 2013). Adoption of this method has been encouraged by the development of a variety of computational tools to assist researchers in conducting these analyses.

2.2.3 Tools for Probing

Probing an interaction by hand is often computationally intense and allows for many opportunities for mistakes and rounding errors. A number of researchers have created tools which take the computational burden, and potential for error, off of the researcher. The first computational tool available for the Johnson-Neyman procedure was developed in the language TELCOMP (Carroll & Wilson, 1970). With input summary statistics, the program could solve for the region of significance in a two-group two-moderator problem, boasting a run time of a mere half hour. Code for computing the Johnson-Neyman points for a dichotomous independent variable

in both SPSS and BDMP was provided in Karpman (1983) and expanded to SAS in Karpman (1986). Pedhazur (1997) and O'Connor (1998) provided programs compatible for SPSS and SAS which computed simple-slopes analysis for two- and three-way interactions. Preacher et al. (2006) provide an online tool which takes a variety of inputs generated from a traditional statistical package and can output both simple-slopes and Johnson-Neyman solutions for multiple linear regression, hierarchical linear models, and latent curve analysis. The first within-package tool for SPSS and SAS which could compute both simple-slopes and the Johnson-Neyman procedure for continuous and dichotomous outcome variables was MODPROBE (Hayes & Matthes, 2009). Most of the capabilities of MODPROBE have since been integrated into PROCESS, a tool for SPSS and SAS which estimates moderation, mediation, and conditional process models (Hayes, 2013).

Probing interactions is an important part of understanding how an the effect of the independent variable on an outcome looks and behaves along the range of the moderator. Methods for probing interactions (simple-slopes and the Johnson-Neyman procedure) as well as accompanying tools for these methods have been available for a number of years. Since there has not previously been a method for implementing the Johnson-Neyman technique with categorical independent variables, I will provide a tool to conduct the analysis, reducing the burden on the researcher.

Chapter 3: Moderation of the Effect of a Categorical Variable

There are many instances where researchers are interested in moderation and the predictor of interest X is categorical, such as race or religion or experimental condition (when there are more than two conditions). For example, Barajas-Gonzales and Brooks-Gunn (2014) investigated the relationship between participants' ethnicity (White, Black, or Latino) and fear of safety in their neighborhood. They proposed that some ethnic groups may be more reactive to neighborhood disorder than other groups, resulting in an interaction between ethnicity and neighborhood disorder in predicting fear for safety. In a different study, Niederdeppe, Shapiro, Kim, Bartolo, and Porticella (2014) had participants read one of three narratives about a woman's experience with weight loss, where each story varied how much personal responsibility she took for her inability to lose weight (categorized as low, moderate, and high). Participants then indicated their support for a variety of government policies which might help individuals lose weight (e.g., increasing sidewalks in neighborhoods). They found that story narrative had essentially no effect among those high in liberal beliefs, but individuals low in liberal beliefs were more supportive of policies when the woman in the story took low or moderate responsibility for her weight loss. Many other examples of moderation of the effect of a categorical variable can be found throughout

psychology and other social sciences (e.g., Cleveland et al., 2013; O'Malley, Voight, Renshaw, & Eklund, 2015).

In this chapter I describe how to make inference about moderation when X is categorical, focusing particularly on the case where X has three or more categories. Just as in the case of a continuous or dichotomous X, probing a moderation effect is key to understanding how the effect of X on Y changes across the range of M. I will describe the currently available methods for probing these types of interactions in this chapter, leaving the development of the Johnson-Neyman procedure for the following chapter.

3.1 Inference about Moderation

In the categorical case, X can be represented in linear regression using $k-1$ variables, where k is the number of categories in X. There are a number of ways to code X into these new variables, one of the most popular of which is dummy coding (also known as indicator coding). Dummy coding is a method which recodes a categorical variable into $k-1$ dichotomous variables which take the value of either 0 or 1 depending on which group the case is in. Each of the $k-1$ variables corresponds to a specific group in X, with one group lacking a corresponding variable. If participant i is in group j then the dummy variable corresponding to group j will equal 1 and all other dummy variables will equal zero for case i. The one group which does not have a corresponding dummy variable is often referred to as the *reference group*, and individuals in this group have scores of zero on all dummy variables. An example of dummy coding is provided below, where D_1 corresponds to participants in Group 1,

D_2 corresponds to participants in Group 2, D_3 corresponds to participants in Group 3, and participants in Group 4 are the reference group.

X	D_1	D_2	D_3
1	1	0	0
2	0	1	0
3	0	0	1
4	0	0	0

I will continue throughout the manuscript under the assumption that dummy codes are being used to describe the categorical variable of interest. However, any other kind of coding can be used without loss of generality.

As in the case of two groups, researchers interested in testing questions of moderation can set up two competing models, one model where the effect of X (now coded in the D variables) is not contingent on some moderator M and another model where the effect of X is contingent on M. Let us consider the example of three groups. Because the effect of X is now captured by 2 variables (D_1 and D_2), the effect of each of these variables should be allowed to be contingent on M as such:

$$\text{Model 1: } Y_i = b_0^* + b_1^* D_{1i} + b_2^* D_{2i} + b_3^* M_i + \epsilon_i^*$$

$$\text{Model 2: } Y_i = b_0 + \Theta_{D_1 \rightarrow Y|M} D_{1i} + \Theta_{D_2 \rightarrow Y|M} D_{2i} + b_3 M_i + \epsilon_i$$

$$\Theta_{D_1 \rightarrow Y|M} = b_1 + b_4 M_i$$

$$\Theta_{D_2 \rightarrow Y|M} = b_2 + b_5 M_i$$

Here the effect of D_1 and the effect of D_2 are linear functions of M. By plugging in the equations for $\Theta_{D_1 \rightarrow Y|M}$ and $\Theta_{D_2 \rightarrow Y|M}$ and expanding, the equation for Model 2 can be re-expressed as:

$$Y_i = b_0 + b_1 D_{1i} + b_2 D_{2i} + b_3 M_i + b_4 M_i D_{1i} + b_5 M_i D_{2i} + \epsilon_i \tag{3.1}$$

From Equation 3.1 it is clear that Model 1 is nested under Model 2, in that if both b_4 and b_5 are zero, then Model 2 is equivalent to Model 1. Most regression software does not provide results for simultaneous inference about multiple coefficients in the model, but rather provides inferences about each coefficient on its own. However, similar to the two group case, hierarchical regression analysis can be used to test if adding the product terms D_1M and D_2M to Model 1 explains additional variance in Y (i.e., if the joint hypothesis that both b_4 and b_5 are zero can be rejected). This type of analysis can be completed using most statistical packages which can estimate linear regression models. If the product terms explain a significant portion of additional variance as assessed by applying Equation 2.1 and associated hypothesis tests, then this is evidence that the relationship between X and Y is indeed contingent on M.

If there are more than three groups, additional dummy coded variables are needed, and thus additional conditional relationships will be needed to fully quantify the conditional relationship between X and Y. For example, Model 2 for a four group case would be written as such:

$$\text{Model 2: } Y_i = b_0 + \Theta_{D_1 \to Y|M} D_{1i} + \Theta_{D_2 \to Y|M} D_{2i} + \Theta_{D_3 \to Y|M} D_{3i} + b_4 M_i + \epsilon_i$$

$$\Theta_{D_1 \to Y|M} = b_1 + b_5 M_i$$

$$\Theta_{D_2 \to Y|M} = b_2 + b_6 M_i$$

$$\Theta_{D_3 \to Y|M} = b_3 + b_7 M_i$$

3.2 Probing Moderation Effects

Just like in the continuous or dichotomous case, a test of moderation is often insufficient for answering all the questions a researcher may pose. For example, Neiderdeppe et al. (2014) may be interested in identifying the range of scores on their

political ideology scale (liberal – conservative) which correspond to significant differences among the story narratives. Methods for probing moderation of the relationship between a categorical independent variable and a continuous outcome have been discussed in some books and publications (e.g., Cohen et al., 2003; Darlington & Hayes, 2017; Spiller et al., 2013), but not nearly as much as in the dichotomous or continuous independent variable case. Specifically, what differentiates the categorical case is that there is not always a single function which can describe the conditional effect of X on Y, but rather $k-1$ functions which must be taken together to describe the conditional effect of X on Y.

3.2.1 Simple-Slopes Analysis

The methods described in previous sections could be used to test pairwise differences between groups at specific values of a moderator. However other methods must be used to test for omnibus group differences at a specific value of the moderator, say m. The test for a dichotomous or a continuous independent variable relies on a single estimate of the conditional effect of X on Y, $\Theta_{X \rightarrow Y|M=m}$, and its estimated standard error, $\hat{s}_{\Theta_{X \rightarrow Y|M=m}}$. However, in the categorical case there are $k-1$ conditional effects, which must be considered all together in order to make a claim about omnibus group differences.

To test the hypothesis of no group differences in Y at a specific value of the moderator, a researcher can set up and compare the fit of two models: one which fixes all of the groups to be equal on Y at the value of interest, m, and one which allows the groups to differ in Y at m. If allowing the groups to differ at m yeilds a

better fitting model of Y, then this supports the claim that the groups vary on Y at m, and thus there is an omnibus effect of X on Y at $M = m$.

To decide how to set up these models, let us examine the interpretations of the regression coefficients in Equation 3.1. The interpretation of b_1 is the predicted change in Y with a one unit change in D_1 when M is zero. When D_1 is a dummy coded variable, this indicates the estimated difference in Y between the group coded with D_1 and the reference group when M is zero. Similarly, when using dummy coding, b_2 is the estimated difference in Y between the group coded with D_2 and the reference group when M is zero. Therefore, when both b_1 and b_2 are zero, there are no group differences when M is zero.

A researcher could use hierarchical regression to test if b_1 and b_2 are both zero by setting up one model which fixes b_1 and b_2 to be zero, and one that allows them to vary.

$$\text{Model 1: } Y_i = b_0^* + b_1^* M_i + b_2^* D_{1i} M_i + b_3^* D_{2i} M_i + \epsilon_i^*$$

$$\text{Model 2: } b_0 + b_1 D_{1i} + b_2 D_{2i} + b_3 M_i + b_4 D_{1i} M_i + b_5 D_{2i} M_i + \epsilon_i$$

From the above equations it is clear that Model 1 is nested within Model 2, where if b_1 and b_2 both equal zero in Model 2 then Model 2 is the same as Model 1. Using hierarchical regression, estimate Model 1 then Model 2. If Model 2 explains significantly more variance in Y than Model 1, this is evidence that b_1 and b_2 are not both equal to zero, and thus there are group differences at the point where M is equal to zero.

Based on this method it is easy to probe the effect of X and Y when $M = 0$. Instead, we would like a general method for probing at any value of M, not just

$M = 0$. In order to probe the effect of X on Y at any point along M, say m, a researcher should center the variable M at m, call this new variable $M^c = M - m$ and use the same hierarchical regression method as above.

$$\text{Model 1: } Y_i = b_0^* + b_1^* M_i^c + b_2^* D_{1i} M_i^c + b_3^* D_{2i} M_i^c + \epsilon_i^*$$

$$\text{Model 2: } b_0 + b_1 D_{1i} + b_2 D_{2i} + b_3 M_i^c + b_4 D_{1i} M_i^c + b_5 D_{2i} M_i^c + \epsilon_i$$

It is now clear how to test for group differences in Y at any value of the moderator. This method is equivalent to the simple-slopes method for two conditions, and would result in the same conclusions as the method described above if used for a dichotomous predictor X.

Though intuitive to some, it may be more clear to explain *why* re-centering works. This can be described in the form of a model comparison, where one model fixes the group differences in $Y$to be zero at m and the other allows the groups to vary at m. The unconstrained model does not depend on the value of m chosen. However, by beginning with the unconstrained model it is possible to derive the constrained model in a general form, showing why the re-centering strategy proposed above works. The unconstrained model for three groups can be described as such:

$$Y_i = b_0 + \Theta_{D_1 \to Y|M} D_{1i} + \Theta_{D_2 \to Y|M} D_{2i} + b_3 M_i + \epsilon_i \tag{3.2}$$

$$\Theta_{D_1 \to Y|M} = b_1 + b_4 M_i$$

$$\Theta_{D_2 \to Y|M} = b_2 + b_5 M_i$$

Because the question of interest is if the effect of X on Y is zero at m, constrain both $\Theta_{D_1 \to Y|M=m}$ and $\Theta_{D_2 \to Y|M=m}$ to be zero.

$$0 = b_1 + b_4 m$$

$$0 = b_2 + b_5 m$$

This implies:

$$b_1 = -b_4 m$$

$$b_2 = -b_5 m$$

Plugging this constraint into Equation 3.2 gives:

$$Y_i = b_0 - b_4 m D_{1i} - b_5 m D_{2i} + b_3 M_i + b_4 D_{1i} M_i + b_5 D_{2i} M_i + \epsilon_i$$

Reordering and grouping terms results in

$$Y_i = b_0 + (M_i - m) b_4 D_{1i} + (M_i - m) b_5 D_{2i} + b_3 M_i + \epsilon_i$$

From this equation it is clear how the re-centering method described earlier empirically tests the omnibus group differences at a specific value of $M = m$. Formal derivations of this method, the model sums of squares, and hypothesis tests involved for any number of groups can be found in Forster (1971, 1974).

The next extension would be to ask if a range of the moderator could be defined such that any point along that range would result in rejecting the hypothesis of no group differences. This is the formulation of the Johnson-Neyman procedure with a categorical independent variable, and the primary topic of this thesis.

Chapter 4: Derivations of the Johnson-Neyman Procedure for Multiple Groups

Using an application of the approach to the Johnson-Neyman procedure in linear regression from Bauer and Curran (2005) and the principles of hypothesis tests for sets of regression coefficients, I will derive the boundary of significance for an omnibus test of group difference along some moderator M. I begin with the derivation for three groups and continue with a partial derivation for four groups. The solution for two groups relies on solving for the roots of a two-degree polynomial, achieved easily by applying the quadratic equation. The derivation of the Johnson-Neyman boundary of significance for the three-group case relies on solving for the roots of a fourth-degree polynomial, for which closed form solutions are available. In the four-group derivation, the roots of an eighth-degree polynomial are required. The Abel-Ruffini theorem states that there are no algebraic solutions for the roots of polynomials of degree five or more (Abel, 1824; Ruffini, 1799). To deal with the issue of no closed form algebraic solution for the boundary of significance I provide an iterative computer program that solves for the Johnson-Neyman boundaries for any number of groups.

4.1 Three Groups

The region of significance is the range of the moderator such that any point within that range results in rejecting the null hypothesis that there are no group differences in Y at that point. These points can be described as those where allowing $\Theta_{D_1 \rightarrow Y|M=m}$ and $\Theta_{D_2 \rightarrow Y|M=m}$ to be non-zero explains a significant amount of variance in Y. The test of significance for the increase in variance explained is based on an F statistic which can be calculated using Equation 2.1. This equation can be rewritten using matrix algebra, and in this form I will use it to derive the boundaries of significance in the three-group case.

$$F = \frac{(L'\hat{\beta})(L'\Sigma_{\hat{\beta}}L)^{-1}(L'\hat{\beta})}{q} \tag{4.1}$$

Recall from Chapter 2 that p is the number of predictors in the unconstrained model, and q is the number of constraints made to the unconstrained model to results in the constrained model. In the case of three groups $q = 2$. Here L' is a $q \times (p+1)$ matrix which describes the model constraints under the null hypothesis. $\hat{\beta}$ is a $(p+1) \times 1$ column vector containing the OLS estimates of the regression coefficients from Model 2. $\Sigma_{\hat{\beta}}$ is the estimated variance-covariance matrix of the regression coefficients of size $(p+1) \times (p+1)$.

First consider the original data matrix, X. This matrix is not used in any of the further equations, but it is important to note that formatting the data matrix in this way results in the interpretations of the estimated regression coefficients below matching the equations used above, particularly Equation 4.1.

$$X = \begin{bmatrix} 1 & D_{11} & D_{21} & M_1 & D_{11}M_1 & D_{21}M_1 \\ 1 & D_{12} & D_{22} & M_2 & D_{12}M_2 & D_{22}M_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & D_{1n} & D_{2n} & M_n & D_{1n}M_n & D_{2n}M_n \end{bmatrix}$$

The corresponding regression coefficient estimates from Model 2 would be

$$\hat{\beta}' = \begin{bmatrix} \hat{b}_0 & \hat{b}_1 & \hat{b}_2 & \hat{b}_3 & \hat{b}_4 & \hat{b}_5 \end{bmatrix}$$

Because the null hypothesis is that $\Theta_{D_1 \to Y|M=m} = b_1 + b_4 m = 0$ and $\Theta_{D_2 \to Y|M=m} = b_2 + b_5 m = 0$ our contrast matrix L is defined as

$$L' = \begin{bmatrix} 0 & 1 & 0 & 0 & m & 0 \\ 0 & 0 & 1 & 0 & 0 & m \end{bmatrix}$$

It may not be initially clear why L has been chosen in this manner, but once $L'\hat{\beta}$ is examined, it is clear that the estimates of the functions of interest $\hat{\Theta}_{D_1 \to Y|M=m}$ and $\hat{\Theta}_{D_2 \to Y|M=m}$ are defined by this contrast matrix.

$$L'\hat{\beta} = \begin{bmatrix} \hat{b}_1 + \hat{b}_4 m \\ \hat{b}_2 + \hat{b}_5 m \end{bmatrix}$$

Additionally, because the individual variance and covariance components will be integral to these derivations, $\Sigma_{\hat{\beta}}$ will be defined as

$$\Sigma_{\hat{\beta}} = \begin{bmatrix} v_0 & c_{01} & c_{02} & c_{03} & c_{04} & c_{05} \\ c_{01} & v_1 & c_{12} & c_{13} & c_{14} & c_{15} \\ c_{02} & c_{12} & v_2 & c_{23} & c_{24} & c_{25} \\ c_{03} & c_{13} & c_{23} & v_3 & c_{34} & c_{35} \\ c_{04} & c_{14} & c_{24} & c_{34} & v_4 & c_{45} \\ c_{05} & c_{15} & c_{25} & c_{35} & c_{45} & v_5 \end{bmatrix}$$

Here the estimated sampling variance of each regression coefficient is defined by the variable v with same subscript as the regression coefficient. For example, the estimated sampling variance of $\hat{b}_4$ is v_4. Similarly, the estimated sampling covariance

of two regression coefficients is noted by the variable c with the same subscripts as regression coefficients. For consistency, the smallest subscript is always listed first. For example, the estimated sampling covariance of $\hat{b}_1$ and $\hat{b}_5$ is noted as c_{15}.

Applying Equation 4.1 to the above defined matrices

$$\begin{aligned}(L'\hat{\beta})(L'\Sigma_{\hat{\beta}}L)^{-1}(L'\hat{\beta}) &= \mid L'\Sigma_{\hat{\beta}}L \mid^{-1} [(\hat{b}_1+\hat{b}_4m)[(v_2+2c_{25}m+m^2v_5)(\hat{b}_1+\hat{b}_4m)- \\ &\quad (\hat{b}_2+\hat{b}_5m)(c_{12}+c_{15}m+c_{24}m+c_{45}m^2)]+ \\ &\quad (\hat{b}_2+\hat{b}_5m)[(v_1+2c_{14}m+v_4m^2)(\hat{b}_2+\hat{b}_5m)- \\ &\quad (\hat{b}_1+\hat{b}_4m)(c_{12}+c_{15}m+c_{24}m+c_{45}m^2)]]\end{aligned}$$

Where

$$\mid L'\Sigma_{\hat{\beta}}L \mid = (v_1+2c_{14}m+v_4m^2)(v_2+2c_{25}m+v_5m^2)-(c_{12}+c_{24}m+c_{15}m+c_{45}m^2)^2$$

Plugging in these values to Equation 4.1:

$$F=\frac{(\hat{b}_1+\hat{b}_4m)^2(v_2+2c_{25}m+v_5m^2)+(\hat{b}_2+\hat{b}_5m)^2(v_1+2c_{14}m+v_4m^2)}{2[(v_1+2c_{14}m+v_4m^2)(v_2+2c_{25}m+v_5m^2)-(c_{12}+c_{24}m+c_{15}m+c_{45}m^2)^2]} \tag{4.2}$$

The boundary of significance is defined by values of m such that F in Equation 4.2 is exactly significant as defined by some preset test level α (typically chosen within the range of .01 - .1). The inverse cumulative distribution of F is a function which, given a certain probability p between zero and one, outputs the point along the F distribution such that $100p\%$ of the distribution falls below that point. The critical value of F, F_{crit}, is the value of F such that the statistic is exactly statistically significant at some value α and is the inverse cumulative distribution function of F at α with q and df_2 degrees of freedom (recall that df_2 is the residual degrees of freedom from the unconstrained model). When the F statistic as defined by Equation 4.2

is exactly equal to F_{crit}, this statistic will be exactly significant, and thus values of m such that F as defined in Equation 4.2 is equal to F_{crit} define the boundary of significance.

To find the boundary of significance, plug in F_{crit} and q and solve for m. By plugging in these values, setting the left hand side equal to zero, and reorganizing terms it is clear that this equation is a fourth-degree polynomial in m.

$$\begin{aligned} 0 \;=\; & (\hat{b}_1^2 v_2 + \hat{b}_2^2 v_1 + 2F_{crit} v_1 v_2 c_{12}^2) + \\ & 2[c_{23}\hat{b}_1^2 + \hat{b}_1\hat{b}_4 v_2 + c_{14}\hat{b}_2^2 + \hat{b}_2\hat{b}_5 v_1 + 2F_{crit}(c_{12}c_{24} + c_{12}c_{15} - v_1 c_{25} - v_2 c_{14})]m + \\ & [v_5\hat{b}_1^2 + 4c_{25}\hat{b}_1\hat{b}_4 + \hat{b}_4^2 v_2 + v_4\hat{b}_2^2 + 4c_{14}\hat{b}_2\hat{b}_5 + \hat{b}_5^2 v_1 + \\ & 2F_{crit}(2c_{45}c_{12} + c_{24}^2 + 2c_{24}c_{15} + c_{15}^2 - v_1 v_5 - 4c_{14}c_{25} - v_2^2)]m^2 + \\ & 2[v_5\hat{b}_1\hat{b}_4 + c_{25}\hat{b}_4^2 + v_4\hat{b}_2\hat{b}_5 + c_{14}\hat{b}_5^2 + 2F_{crit}(c_{24}c_{45} + c_{15}c_{45} - v_5 c_{14} - c_{25}v_2)]m^3 + \\ & [v_5\hat{b}_4^2 + v_4\hat{b}_5^2 - 2F_{crit} v_2 v_5 c_{45}^2]m^4 \end{aligned} \tag{4.3}$$

The solutions for the roots of this equation are long algebraic equations. There are four solutions, some of which may be imaginary depending on specific values of the regression coefficients, variances, and covariances. Below is one of the solutions. In order to simplify notation, let each coefficient from Equation 4.3 be equal to some variable.

$$\begin{aligned} d \;=\; & \hat{b}_1^2 v_2 + \hat{b}_2^2 v_1 + F_{crit} 2 v_1 v_2 \\ e \;=\; & 2[c_{23}\hat{b}_1^2 + \hat{b}_1\hat{b}_4 v_2 + c_{14}\hat{b}_2^2 + \hat{b}_2\hat{b}_5 v_1 + 2F_{crit}(c_{12}c_{24} + c_{12}c_{15} - v_1 c_{25} - v_2 c_{14})] \\ f \;=\; & v_5\hat{b}_1^2 + 4c_{25}\hat{b}_1\hat{b}_4 + \hat{b}_4^2 v_2 + v_4\hat{b}_2^2 + 4c_{14}\hat{b}_2\hat{b}_5 + \hat{b}_5^2 v_1 + \\ & 2F_{crit}(2c_{45}c_{12} + c_{24}^2 + 2c_{24}c_{14} + c_{15}^2 - v_1 v_5 - 4c_{14}c_{25} - v_2^2) \\ g \;=\; & 2[v_5\hat{b}_1\hat{b}_4 + c_{25}\hat{b}_4^2 + v_4\hat{b}_2\hat{b}_5 + c_{14}\hat{b}_5^2 + 2F_{crit}(c_{24}c_{25} + c_{15}c_{45} - v_5 c_{14} - c_{25}v_2)] \end{aligned}$$

$$h = v_5\hat{b}_4^2 + v_4\hat{b}_5^2 - 2F_{crit}v_2v_5$$

Using these new variables the solution for one root of Equation 4.3 can be expressed algebraically. This is the first Johnson-Neyman solution, m_{JN_1}. For the sake of brevity, and because these equations would typically be implemented in a computer program, there is no need to express the other roots. They are all of a similar form, based completely off the variables d, e, f, g, and h.

$$\begin{aligned}
M_{JN_1} = & -\frac{g}{4h} + \frac{1}{2}(\frac{g^2}{4h^2} - \frac{2f}{3h} + \frac{1}{6}h(-288dfh + 108dg^2 + 108e^2h - 36efg + 8f^3 + \\
& 12(-768d^3h^3 + 576d^2egh^2 + 384d^2f^2h^2 - 432d^2fg^2h + \\
& 81d^2g^4 - 432de^2fh^2 + 18de^2g^2h + 240def^2gh - 54defg^3 - 48df^4h + \\
& 12df^3g^2 + 81e^4h^2 - 54e^3fgh + 12e^3g^3 + 12e^2f^3h - 3e^2f^2g^2)^{1/2})^{1/3} + \\
& \frac{2}{3}(12dh - 3eg + f^2)/(h(-288dfh + 108dg^2 + 108e^2h - 36efg + 8f^3 + \\
& 12(-768d^3h^3 + 576d^2egh^2 + 384d^2f^2h^2 - 432d^2fg^2h + 81d^2g^4 - \\
& 432de^2fh^2 + 18de^2g^2h + 240def^2gh - 54defg^3 - 48df^4h + 12df^3g^2 + \\
& 81e^4h^2 - 54e^3fgh + 12e^3g^3 + 12e^2f^3h - 3e^2f^2g^2)^{1/2})^{1/3}))^{1/2} + \\
& \frac{1}{2}(\frac{g^2}{2h^2} - \frac{4f}{3h} - \frac{1}{6h}(-288dfh + 108dg^2 + 108e^2h - 36efg + 8f^3 + \\
& 12(-768d^3h^3 + 576d^2egh^2 + 384d^2f^2h^2 - 432d^2fg^2h + 81d^2g^4 - \\
& 432de^2fh^2 + 18de^2g^2h + 240def^2gh - 54defg^3 - 48df^4h + 12df^3g^2 + \\
& 81e^4h^2 - 54e^3fgh + 12e^3g^3 + 12e^2f^3h - 3e^2f^2g^2)^{1/2})^{1/3} - \\
& \frac{2}{3}(12dh - 3eg + f^2)/(h(-288dfh + 108dg^2 + 108e^2h - 36efg + 8f^3 + \\
& 12(-768d^3h^3 + 576d^2egh^2 + 384d^2f^2h^2 - 432d^2fg^2h + 81d^2g^4 - \\
& 432de^2fh^2 + 18de^2g^2h + 240def^2gh - 54defg^3 - 48df^4h + 12df^3g^2 +
\end{aligned}$$

$$81e^4h^2 - 54e^3fgh + 12e^3g^3 + 12e^2f^3h - 3e^2f^2g^2)^{1/2})^{1/3}) +$$
$$(\frac{fg}{h^2} - \frac{2e}{h} - \frac{g^3}{4h^3})/(\frac{g^2}{4h^2} - \frac{2f}{3h} + \frac{1}{6h}(-288dfh + 108dg^2 + 108e^2h -$$
$$36efg + 8f^3 + 12(-768d^3h^3 + 576d^2egh^2 + 384d^2f^2h^2 - 432d^2fg^2h +$$
$$81d^2g^4 - 432de^2fh^2 + 18de^2g^2h + 240def^2gh - 54defg^3 - 48df^4h +$$
$$12df^3g^2 + 81e^4h^2 - 54e^3fgh + 12e^3g^3 + 12e^2f^3h - 3e^2f^2g^2)^{1/2})^{1/3} +$$
$$\frac{2}{3}(12dh - 3eg + f^2)/(h(-288dfh + 108dg^2 + 108e^2h - 36efg + 8f^3 +$$
$$12(-768d^3h^3 + 576d^2egh^2 + 384d^2f^2h^2 - 432d^2fg^2h + 81d^2g^4 -$$
$$432de^2fh^2 + 18de^2g^2h + 240def^2gh - 54defg^3 - 48df^4h + 12df^3g^2 +$$
$$81e^4h^2 - 54e^3fgh + 12e^3g^3 + 12e^2f^3h - 3e^2f^2g^2)^{1/2})^{1/3}))^{1/2})^{1/2}$$

Using the solutions for the roots of quartic equations, the solutions for the Johnson-Neyman boundary of significance for a test of omnibus group differences in the case of three groups are well defined.

Though these equations are notably complicated, they are not too unwieldy to be programmed into a computer program, such as an SPSS or SAS macro or R-package, to solve for the Johnson-Neyman boundaries of significance for linear regression problems with a continuous moderator and a three-group categorical variable. A computer program that implements this solution would be greatly useful to researchers interested in omnibus group differences which are moderated by a continuous variable. These solutions will be able to inform them of the range of the moderator variable which defines significant group differences and non-significant group differences.

Though this is the first time an algebraic solution has been derived for the three-groups case, it would be ideal to provide a general solution for any number of groups.

In order to investigate this as a possibility, I perform a similar derivation using the same equations and an expanded contrast matrix for the four-group case.

4.2 Four Groups

In order to define the Johnson-Neyman boundaries for the four group case, the models to be compared should first be defined. Because there are four groups, the effect of group will be coded into three dummy coded variables, D_1, D_2, and D_3. Model 2 will represent the situation in which the effect of group as represented by the dummy coded variables is allowed to vary as a linear effect of a moderator variable M.

$$\text{Model 2: } Y_i = b_0 + \Theta_{D_1 \to Y|M} D_{1i} + \Theta_{D_2 \to Y|M} D_{2i} + \Theta_{D_3 \to Y|M} D_{3i} + b_4 M_i + \epsilon_i \quad (4.4)$$

$$\Theta_{D_1 \to Y|M} = b_1 + b_5 M_i$$

$$\Theta_{D_2 \to Y|M} = b_2 + b_6 M_i$$

$$\Theta_{D_3 \to Y|M} = b_3 + b_7 M_i$$

Under the null hypothesis, that there are no group differences when $M = m$, i.e. each of $\Theta_{D_1 \to Y|M=m}$, $\Theta_{D_2 \to Y|M=m}$, and $\Theta_{D_1 \to Y|M=m}$ are zero.

$$0 = b_1 + b_5 m$$

$$0 = b_2 + b_6 m$$

$$0 = b_3 + b_7 m$$

This implies

$$b_1 = -b_5 m$$

$$b_2 = -b_6 m$$

$$b_3 = -b_6 m$$

To get Model 1, plug in these contraints to Equation 4.4.

Model 1:

$$Y_i = b_0 + (-b_5 m + b_5 M_i) D_{1i} + (-b_6 m + b_6 M_i) D_{2i} + (-b_7 m + b_7 M_i) D_{3i} + b_4 M_i + \epsilon_i$$

Reordering and grouping terms results in

$$\text{Model 1: } Y_i + b_0 + (M_i - m) b_5 D_{1i} + (M_i - m) b_6 D_{2i} + (M_i - m) b_7 D_{3i} + b_4 M_i + \epsilon_i$$

Constraining the conditional effect of each dummy coded variable on Y to be zero at m results in a model which includes the product of the re-centered M variable and each dummy coded variable as well as the M variable.

To derive the Johnson-Neyman boundaries of significance, apply Equation 4.1 to assess change in model fit, using the new X, $\hat{\beta}$, L, and $\Sigma_{\hat{\beta}}$ matrices specific to the four group case.

$$X = \begin{bmatrix} 1 & D_{11} & D_{21} & D_{31} & M_1 & D_{11}M_1 & D_{21}M_1 & D_{31}M_1 \\ 1 & D_{12} & D_{22} & D_{32} & M_2 & D_{12}M_2 & D_{22}M_2 & D_{32}M_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & D_{1n} & D_{2n} & D_{3n} & M_n & D_{1n}M_n & D_{2n}M_n & D_{3n}M_n \end{bmatrix}$$

$$\hat{\beta}' = \begin{bmatrix} \hat{b}_0 & \hat{b}_1 & \hat{b}_2 & \hat{b}_3 & \hat{b}_4 & \hat{b}_5 & \hat{b}_6 & \hat{b}_7 \end{bmatrix}$$

$$L' = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & m & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & m & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & m \end{bmatrix}$$

$$\Sigma_{\hat{\beta}} = \begin{bmatrix} v_0 & c_{01} & c_{02} & c_{03} & c_{04} & c_{05} & c_{06} & c_{07} \\ c_{01} & v_1 & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & c_{17} \\ c_{02} & c_{12} & v_2 & c_{23} & c_{24} & c_{25} & c_{26} & c_{27} \\ c_{03} & c_{13} & c_{23} & v_3 & c_{34} & c_{35} & c_{36} & c_{37} \\ c_{04} & c_{14} & c_{24} & c_{34} & v_4 & c_{45} & c_{46} & c_{47} \\ c_{05} & c_{15} & c_{25} & c_{35} & c_{45} & v_5 & c_{56} & c_{57} \\ c_{06} & c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & v_6 & c_{67} \\ c_{07} & c_{17} & c_{27} & c_{37} & c_{47} & c_{57} & c_{67} & v_7 \end{bmatrix}$$

Based on these equations, the product of L' and $\hat{\beta}$ define the model constraints of interest.

$$L'\hat{\beta} = \begin{bmatrix} \hat{b}_1 + \hat{b}_5 m \\ \hat{b}_2 + \hat{b}_6 m \\ \hat{b}_3 + \hat{b}_7 m \end{bmatrix}$$

Applying Equation 4.1 to the above defined matrices

$(L'\hat{\beta})'(L'\Sigma_{\hat{\beta}}L)^{-1}(L'\hat{\beta}) =$

$| L'\Sigma_{\hat{\beta}}L |^{-1} [(\hat{b}_1 + \hat{b}_5 m)^2[(v_2 + 2c_{26}m + v_6m^2)(v_3 + 2c_{37}m + v_7m^2) - (c_{23} + c_{36}m + c_{27}m + c_{67}m^2)^2] - (\hat{b}_1 + \hat{b}_5 m)(\hat{b}_2 + \hat{b}_6 m)[(c_{12} + c_{25}m + c_{16}m + c_{56}m^2)(v_3 + 2c_{37}m + v_7m^2) + (c_{13} + c_{35}m + c_{17}m + c_{57}m^2)(c_{23} + c_{27}m + c_{36}m + c_{67}m^2)] + (\hat{b}_1 + \hat{b}_5 m)(\hat{b}_3 + \hat{b}_7 m)[(c_{12} + c_{25}m + c_{16}m + c_{56}m^2)(c_{23} + c_{36}m + c_{27}m + c_{67}m^2) - (c_{13} + c_{25}m + c_{17}m + c_{57}m^2)(v_2 + 2c_{26}m + v_6m^2)] - (b_1 + b_5 m)(b_2 + b_6 m)[(c_{12} + c_{25}m + c_{16}m + c_{56}m^2)(v_3 + 2c_{37}m + v_7m^2) + (c_{13} + c_{35}m + c_{17}m + c_{57}m^2)(c_{23} + c_{27}m + c_{36}m + c_{67}m^2)] + (\hat{b}_2 + \hat{b}_6 m)^2[(v_1 + 2c_{15}m + v_5m^2)(v_3 + 2c_{37}m + v_7m^2) - (c_{13} + c_{35}m + c_{17}m + c_{57}m^2)^2] + (\hat{b}_2 + \hat{b}_6 m)(\hat{b}_3 + \hat{b}_7 m)[(c_{13} + c_{35}m + c_{17}m +$

$$c_{57}m^2)(c_{12} + c_{16}m + c_{25}m + c_{56}m^2) - (v_1 + 2c_{15}m + v_5m^2)(c_{23} + c_{36}m + c_{27}m +$$
$$c_{67}m^2)] + (\hat{b}_1 + \hat{b}_5m)(\hat{b}_3 + \hat{b}_7m)[(c_{12} + c_{25}m + c_{16}m + c_{56}m^2)(c_{23} + c_{36}m + c_{27}m +$$
$$c_{67}m^2) - (c_{13} + c_{35}m + c_{17}m + c_{57}m^2)(v_2 + 2c_{26}m + v_6m^2)] + (\hat{b}_2 + \hat{b}_6m)(\hat{b}_3 +$$
$$\hat{b}_7m)[(c_{13} + c_{35}m + c_{17}m + c_{57}m^2)(c_{12} + c_{16}m + c_{25}m + c_{56}m^2) - (v_1 + 2c_{15}m +$$
$$v_5m^2)(c_{23} + c_{36}m + c_{27}m + c_{67}m^2)] + (\hat{b}_3 + \hat{b}_7m)^2[(v_1 + 2c_{15}m + v_5m^2)(v_2 +$$
$$2c_{26}m + v_6m^2) - (c_{12} + c_{25}m + c_{16}m + c_{36}m^2)^2]]$$

Where

$$\begin{aligned} \mid L'\Sigma_{\hat{\beta}}L \mid \;=\; & (v_1 + 2mc_{15} + v_5m^2)(v_2 + 2c_{26}m + v_6m^2)(v_3 + 2c_{37}m + v_7m^2) - (v_1 + \\ & 2c_{15}m + v_5m^2)(c_{23} + c_{36}m + c_{27}m + c_{67}m^2)(c_{23} + c_{27}m + c_{36}m + \\ & c_{67}m^2) - (c_{12} + c_{25}m + c_{16}m + c_{56}m^2)^2(v_3 + 2c_{37}m + v_7m^2) + 2(c_{12} + \\ & c_{25}m + c_{16}m + c_{56}m^2)(c_{23} + c_{36}m + c_{27}m + c_{67}m^2)(c_{13} + c_{17}m + \\ & c_{35}m + c_{57}m^2) - (c_{13} + c_{17}m + c_{35}m + c_{57}m^2)(v_2 + 2c_{26}m + v_6m^2) \end{aligned}$$

Again, F is defined as a polynomial function of m as in the three condition case. A polynomial for which the roots would determine the Johnson-Neyman boundary of significance can be defined by setting F to its critical value given the degrees of freedom in this problem, and setting one side of the equation to zero. In doing this (though excluded for the sake of space) this polynomial is an eighth degree polynomial in m. By the Abel-Ruffini theorem (1824) there is no closed form algebraic solution for the roots of this equation, thus precluding the derivation of the solutions for the Johnson-Neyman boundary of significance. Hunka (1995) and Hunka and Leighton (1997) proposed the use of *Mathematica* to apply these matrix calculations given a specific data set. Their examples, however, only examined up to three-groups,

and *Mathematica* cannot calculate the roots of all equations of degree five or higher ("Roots", n.d.). This means that the methods proposed by Hunka and colleagues are limited to three-groups or fewer.

Without a method for finding the Johnson-Neyman boundary of significance in the four-condition case, it may seem that a solution for finding these boundaries in a general number of groups is far out of reach. However, it is possible to probe interactions between continuous variables and categorical variables of any number of categories using the simple-slopes method. A computer program could repeatedly probe the effect of some categorical variable, honing in on the point at which group differences in Y are exactly significant, thus defining the Johnson-Neyman region of significance without a closed-form solution. For my thesis I developed such a tool, available in two popular statistical packages to increase the potential user base of the tool.

Chapter 5: OGRS: An Iterative Tool for Finding Johnson-Neyman Regions of Significance for Omnibus Group Differences

OGRS (Omnibus Groups Regions of Significance) is an easy to use tool which can probe interactions between a categorical independent variable and a continuous moderator. It is available for two popular statistical packages, SPSS and SAS. After executing the OGRS macro, users will be able to specify a single OGRS command line that specifies all the information needed to do the analyses, while requiring no mathematics on the part of the user. The tool will produce typical regression output, the Johnson-Neyman boundaries of significance, and a table which describes how the effect of the independent variable changes across the observed range of the moderator. See Appendix A and B for SPSS code and documentation, and Appendix C and D for SAS code and documentation.

5.1 Program Inputs

Each language has a different syntax structure for the OGRS command line, but the required inputs are the same across both the SPSS and SAS versions. The only exception is that SAS requires a data file name, whereas SPSS assumes that the active dataset is the one being analyzed. The only required inputs are the variables involved

in the analysis. Optional inputs include confidence level, convergence criteria, and number of initial iterations in the Johnson-Neyman algorithm.

5.1.1 Required Inputs

OGRS requires only one variable as the independent variable in the subcommand `X`. Researchers should save their independent variable as one variable with each group having a unique code (e.g., 1 = Protestant, 2 = Catholic, 3 = Jewish, etc). OGRS recodes this variable into $k - 1$ dummy codes internally for use in regression. Only one variable each will be accepted as input for the moderator and for the outcome variable. Additional covariates can also be included by specifying them in the `vars` command, but not assigning them to any specific role (`X`, `Y`, or `M`). There is no limit to the number of covariates allowed in the model.

Below are examples of the base command line for each language.

```
SPSS   OGRS  vars = var1 var2 var3 var4 var5 /X = var1 /M = var2
             /Y = var3.
SAS   %OGRS  (data = datafile, vars = var1 var2 var3 var4 var5,
              X = var1, M = var2, Y = var3);
```

The list of variables in the `vars` subcommand, specifies all the variables that are used in the regression. Including this command allows researchers to specify additional covariates that do not play the role of independent variable, moderator, or outcome.

5.1.2 Optional Inputs

A few options have been built into OGRS to increase its flexibility and allow users to troubleshoot issues with the Johnson-Neyman algorithm. Researchers can specify the level of confidence, the convergence criteria used by the Johnson-Neyman

algorithm, the number of initial iterations in the Johnson-Neyman algorithm, and the number of decimal places printed in the output. Each of these options has a default value that can be overridden by specifying the name of the subcommand then an equals sign and the new value which is desired (e.g., `CONF = 92`).

Confidence Level

Confidence level is used in two parts of the OGRS routine. In the regression output, confidence intervals are provided alongside each of the estimated regression coefficients. The confidence level specified in the OGRS command line is used to determine the level of confidence at which these intervals are calculated. The default is 95. The users can specify any confidence level greater than 50 and less than 100 in the `CONF` subcommand. The second part of the OGRS routine which uses confidence level is the Johnson-Neyman algorithm. The Johnson-Neyman algorithm searches for the point along the continuous range of the moderator at which the effect of the independent variable on the outcome variable is exactly *statistically significant*. This significance level is determined by the confidence level specified in the `CONF` subcommand. For example, when the confidence level is set at 90, then the p-value corresponding to the effect of the independent variable on the outcome variable at the Johnson-Neyman boundary of significance will be .10. Similarly if the confidence level is specified to be 99, the p-value will be .01.

Convergence Criteria

The convergence criteria is used to calibrate how close the Johnson-Neyman algorithm gets to the exact answer. The default is as precise as the language is capable: .00000001 (up to eight decimal places in both SPSS and SAS). However, if a researcher

is not particularly concerned with the exactness of the solution, then they can specify a more relaxed convergence criteria.

The convergence criteria means different things in each of the languages. In SAS, OGRS looks to converge to a critical F statistic, but in SPSS, OGRS converges to an α level (reasons for this are discussed in the next section). The statistic to which the routine aims to converge, whether it be an F statistic or an α value, will be called the *criterion statistic*, and the desired value of that statistic will be called the *criterion value*. In each of the languages, the default is to find a point that has a corresponding criterion statistic which is within eight decimal places of the criterion value. By specifying a different value in the `CONVCRIT` subcommand, a solution that is that distance (or a smaller distance) from the criterion value will be deemed acceptable. For example if a user specified `CONVCRIT = .0001` then a solution within four decimal points of the criterion value would be acceptable. This can be useful for reducing runtime, as will be discussed later, or if the measurement scale of the variable is not particularly precise, so that the solution does not need to be particularly precise.

Initial Iterations

As discussed in Section 5.5, the Johnson-Neyman algorithm begins by dividing the range of the moderator into sections. The number of sections the space is divded into is determined by the `iter` subcommand. The default for the number of sections is $50 + k * 10$, where k is the number of groups in the independent variable. As will be discussed later, there is a trade off between speed of the program and finding all Johnson-Neyman solutions. Researchers who are particularly concerned about making sure they find all the solutions, but do not care about how long it takes the

program to run, might consider setting the `iter` subcommand to a large number (e.g., 10000).

Decimals

The `DECIMALS` subcommand can be used to specify how many decimal places are printed in the output. The default for this setting is `F10.4` in SPSS and `10.4` in SAS. This argument sets both the number of characters used to represent the number (as set by the number before the decimal point in the subcommand) and the number of decimal places to display to the right of the decimal point (as set by the number after the decimal point in the subcommand). For example `DECIMALS = F8.4` would specify that up to eight characters should be used to represent a number, and the number should display up to four places to the right of the decimal point.

5.1.3 Command Line Example

Below is an example of the OGRS command line using all the different subcommands.

```
SPSS   OGRS  vars = var1 var2 var3 var4 var5 /X = var1
              /M = var2 /Y = var3 /conf = 80 /convcrit = .000001
             /iter = 1000 /decimals = F10.2.
SAS   %OGRS  (data = datafile, vars = var1 var2 var3 var4 var5,
              X = var1, M = var2, Y = var3, conf = 80,
             convcrit = .000001, iter = 1000, decimals = F10.2);
```

In this command, `var1` is the independent variable and dummy coded into $k - 1$ variables, where k is the number of groups in `var1`. The moderator and outcome variables are `var2` and `var3`, respectively. Two covariates, `var4 var5`, are included in the model. All confidence intervals will be 80% confidence intervals, and the Johnson-Neyman algorithm will find points along the moderator where the effect of

the independent variable on the outcome is exactly statistically significant at $\alpha = .20$. Acceptable solutions for the algorithm will be within .000001 of the criterion value, whether it is an F-statistic in SAS or a p-value in SPSS. Initially, the algorithm will divide the range of the moderator into 1000 sections, which will be used to find the solutions, as described below. All output will be reported up to two decimal places.

5.2 Internal Processes

The OGRS routine uses the information provided in the command line to calculate all the information needed to create the output. For example, OGRS can detect the number of groups in the variable input in the `X` subcommand. This will imply the number of groups, which will be important throughout. Additionally the range of the moderator will be defined by the variable input in the `M` subcommand. The iterative approach to the Johnson-Neyman procedure will only search within the observed range of the moderator, as this is the only well-defined space where the researcher has measurements.

5.2.1 Regression Results

After recoding the variable specified in the `X` subcommand, the program will estimate the regression model using all the variables in the `vars` subcommand with the exception of the variable in the `Y` subcommand, to predict the variable specified in the `Y` subcommand, including all product terms between the M variable and the codes representing `X`. All regression coefficients, standard errors, and inferential statistics including t and p-values and confidence intervals are computed and appear in the output. Additionally, a section of output at the bottom of the regression results is dedicated to the test of interaction, comparing the contingent (including all product

terms between the dummy codes for X and M) and non-contingent models (not including any of the product terms).

5.2.2 Finding Johnson-Neyman Solutions

Since it is not possible to directly solve for M such that the associated F statistic reflecting group differences is exactly significant, an alternative is to search for values of M such that this is true. An iterative method to examine a variety of values of M can be used to complete this task. The method for finding the Johnson-Neyman boundaries of significance is based on the bi-section method, a popular method in computer science for finding roots of polynomials.

The Bi-Section Method

The bi-section method is a method for iteratively searching for the point along a closed range where some continuous function equals a prespecified value. In this case we are looking at the function which determines either the p-value associated with the test of the effect of X on Y at different M values (SPSS), or the associated F statistic (SAS). Either way, these functions are both continuous.

The intermediate value theorem states that if the values of this function at the boundaries of the range of the domain of the function span the criterion value, then there is some point where the function is exactly the criterion value within that range. In our case, if we can find two values of the moderator where the criterion statistic is below the criterion value, and another value of the moderator where the criterion statistic is above the criterion value, then somewhere in between those points, the F statistic is exactly the criterion value. This is the principle upon which the bi-section method is based.

The bi-section method assumes that you start with a continuous function where the range of the domain is such that the value of the function at the minimum of the domain is less than (or greater than) the criterion value, and the value of the function at the maximum of the domain is greater than (or less than) the criterion value value. Under this assumption, the bi-section method continuously divides the space in half, evaluating the function at the divided point, then chooses a half which still spans the criterion value until a point which is close enough the criterion value is found.

In the Johnson-Neyman algorithm, we cannot ensure that we have two points which span the criterion value (as there may be no such points), so we divide the space up in small sections, increasing the likelihood that we find two points which span the criterion value, if such points exist. The algorithm then repeatedly applies the bisection method to find the Johnson-Neyman boundaries of significance.

Implementation in OGRS Routine

Using the results from the regression analysis, all of the matrices used in Equation 4.1 are completely known with the exception of L, which contains an unknown m. To search the space along the moderator's range, a jump parameter is specified that divides the range of the moderator. The jump parameter is the width of jump required to span the space of the moderator in a set number of jumps equal to the `iter` subcommand, where the default is $50 + 10 * k$, where k is the number of groups in the independent variable.

The program will identify the range of the moderator and divide this range into sections, solving for the statistical significance of the group differences in Y at each endpoint of the sections. The program will then identify specific sections that transition between significance and non-significance (i.e. sections where one endpoint has

a criterion statistic which is greater than the criterion value and one endpoint has a criterion statistic which is less than the criterion value). These are the sections within which the intermediate value theorem apply, and thus the bi-section method is guaranteed to find a solution. Each section which qualifies is broken up again in a similar manner, solving for the criterion statistic at each of the endpoints of the new sections. This process will repeat until each potential area of transition has resulted in a solution which is close enough to the criterion statistic (F in SAS and α in SPSS), as defined by the convergence criteria. The criterion value α is defined by the `CONF` subcommand, where $\alpha = 1 - \texttt{CONF}/100$. This program will sophisticatedly search the space along the moderator, rather than inefficiently searching the entire space at an overly fine precision.

The program begins by defining a matrix where each row represents a point along the moderator M. The distance between these points is determined by the jump parameter. At each point, the significance of the group differences is evaluating using Equation 4.1, providing an F-statistic reflecting the degree of difference in the groups on the outcome variable at that point on the moderator variable. Depending on the program, either the F statistic is compared to the critical F with $k-1$ and $N-p-1$ degrees of freedom at the α level, or the F statistic is used to compute a p-value which is then compared to the α level.

The SAS version of OGRS tests convergence based on a critical F statistic. F statistics are less variable in order of magnitude than p-values and so the convergence criteria means a similar degree of misfit regardless of the criterion F statistic used. Calculation of the critical F to which the program converges requires a function for the inverse cumulative distribution function of the F distribution. This is something

that SAS has built into PROC IML, which is the language used to program OGRS. However, the SPSS matrix language does not have a similar function. I investigated a few approximations to this function (Abamowitz & Stegun, 1964; Bratley, Fox, & Schrage, 1983); however, the approximations were off by enough that it seemed more reasonable to use the α value instead of the approximate critical-F.

Areas of transition are identified by tagging (in an additional column of the matrix) points where the criterion statistc was less than the specified criterion value in the row above and is now greater than the specified criterion value, or rows where the row above was greater than the criterion value and is now less than the criterion value (i.e. the statistic has transitioned from significant to non-significant or non-significant to significant). The program then checks if either of the two rows involved in the transition converge based on the convergence criteria, and if either does, it does not investigate this area further. When neither of these points is close enough to converge, the program will examine this space on the moderator in more detail.

When a transition area is identified, the program identifies the boundaries of M in this area and divides this range using the same number of jumps as before. Within this space all previous calculations are repeated (Using Equation 4.1 to calculate an F-statistic, and p-value) for each point. This new matrix of results is then inserted into the original results matrix, and the program continues to look for other areas of transition.

Areas are repeatedly searched with greater and greater precision until a solution which is close enough to the criterion value based on the convergence criteria is found. After this solution is found, the program will move on to a new transition area, if one exists. Depending on the number of groups, the number of potential transition

areas will be limited (with only two potential transitions in 2 group, and 4 potential transitions in 3 groups, etc). The program will continue until each area of transition has a subsequent solution. There is currently no maximum number of times the program can divide up a specific space within OGRS.

5.3 Program Outputs

Figure 6.1 provides an example of the output from OGRS for SPSS. The program output includes regression results, Johnson-Neyman results, and a table which shows how the effect of the independent variable on the outcome changes across the moderator. The output begins with a section specifying the variables in the analyses, including X, M, Y, and any covariates. There is also a table which shows how X was dummy coded into $k-1$ variables. Next there is a section with the regression results. This includes all the regression coefficient estimates, standard errors, t-statistics, p-values, and confidence intervals. At the end of the regression results section is the results of comparing the model with no interactions to the model with interactions, and the associated test of significance which is a test of moderation.

The Johnson-Neyman results are presented with a table. The points of transition are printed above the table. After the transition points, a table of other points is provided to give the users a sense of the trends in the change in R^2 and the associated F statistic across the range of M. Approximately 20 lines are printed in the table. This table could be used to graph the change in R^2 or the associated F-statistic across the range of the moderator in order to describe how the effect of the independent variable on the outcome variable changes across this range.

5.4 Programming Decisions

Many programming decisions were made through testing and limitations of languages used. I will first outline the major decisions, then overview some of the testing processes and performance outcomes which drove these decisions.

One of the major concerns was how the program would identify when there are no Johnson-Neyman solutions. OGRS probes the initial points as determined by the `iter` subcommand, and if these points do not identify any transition points, then the program is complete and prints a message which states there are no Johnson-Neyman solutions. During testing, most solutions were identified even with a very small number of initial iterations. In most tested cases, all solutions were identified using only three iterations. However, some specific cases emerged where two solutions were very close together, and thus were not identified with a small number of iterations. The largest number of iterations that did not find all solutions in all test cases was 30. Additionally, the number of possible solutions increases as the number of groups increases. This led to the decision to set the default number of iterations to $50 + 10k$. This would allow for well over 30, which was the minimally sufficient case in testing. Additionally, as will be seen in the run-time performance results, having too many iterations caused the program to run quite slowly. It is very difficult to choose a default for the number of iterations such that there are enough to identify all transition points but not so many as to cause the program to run slowly. This led me to add in the option for researchers to change the number of iterations. That way if the program is running particularly slowly, they can decrease the number of iterations, or if they are concerned that the algorithm missed some solutions, they can increase the number of iterations.

When a transition point is identified and the area is magnified, it is possible to have multiple converging answers all adjacent to each other occur. Essentially, in this case the program has magnified the space so much that many of the points converge. This happened fairly frequently because the distance of the jump parameter gets smaller and smaller as an area is magnified repeatedly. In order to deal with multiple converging answers, the program searches for the two answers which straddle the criterion value (one above and one below) and chooses the closest (with respect to the criterion statistic) of these two points, as they will be the closest overall of the set of converging answers.

As discussed above, the value to which the program converges is either the F-statistic (SAS) or the α level. The F-statistic is preferable, because regardless of the data, the order of magnitude of an F-statistic stays in a fairly limited range (about 1 – 10). An α level though often varies from $.1 - .001$, which spans three orders of magnitude compared to one order of magnitude of the F-statistic. This is an issue when dealing with a convergence criteria. An F-statistic being off by .001 is a roughly comparable error regardless of the the value of the critical F, but a p-value being off by .001, when the critical α is .0001 is a very large error compared to when the critical α is .05. All the approximations which were tested for the inverse cumulative distribution function for the F distribution to be implemented in the SPSS version of OGRS were off by .1 - .3 which can be up to 4% of the F-distribution. For this reason the α value was chosen as the criterion statistic for the SPSS version of OGRS, but the default convergence criteria was set as low as possible (.00000001). This will help to avoid large errors even with fairly large confidence levels. Based on testing,

convergence criteria influenced run time, but not drastically, so it seemed reasonable to set this value to be very low.

Next, I will discuss some of the program tests which were completed in order to understand the limitations of OGRS as well as make some of the programming decisions above.

5.5 Program Performance

There were three major aspects of the program that I wanted to test. The first two are accuracy related: 1) When does the program miss certain answers or get answers that are incorrect, and 2) How accurate are the answers that the program gets. The third aspect of the program is understanding how long it takes to run in a variety of circumstances and understanding what influences how long the program runs.

5.5.1 Accuracy

Perhaps the most important characteristic of an approximation is that is it accurate enough to be useful. Though I cannot decide for potential users whether or not this method is accurate *enough* to be useful, I can provide some information about how accurate this method is, and potential users can decide whether this seems accurate enough.

Accuracy comes in two forms with this algorithm. The algorithm must identify the correct transition points (and not points that are not transition points) and it must estimate those transition points well. Different parts of the algorithm come into play for these two different types of accuracy so I will treat them separately.

Finding Solutions

Finding the correct solutions is determined by whether or not the initial number of iterations is large enough to ensure there are points spanning each transition point. Having these spanning points is what ensures the bi-section method will work, but there is no way to know for sure if all transition points have been identified. If two solutions are contained within one region of the initial division of the range of the moderator, then those solutions will not be found. Because of this it is important to have a sufficient number of initial iterations.

To test how many iterations seems sufficient, I used datasets with 3, 5, and 7 groups, sample sizes of 20, 100, and 1000, and covergence criteria of 10^{-8} and 10^{-4}, and iterations of 3, 4, 5, 10, 20, 30, 40, 50, 100, 1000, and 10000. These datasets were also used to test runtimes. In Tables 5.2 to 5.4 there are rows with asterisks which indicate the iteration numbers which failed to find the correct Johnson-Neyman boundaries of significance. As can be seen from Tables 5.2 to 5.4 the largest value which failed to find the correct Johnson-Neyman boundary in any of the conditions was 30. Though, this may be an artifact of the data generation procedure, and there is no true lower bound on how close two Johnson-Neyman boundaries could be. So there is no way to know completely that all solutions have been found, regardless of the number of initial iterations selected.

Because 30 was the upper bound of the number of iterations which did not find all Johnson-Neyman boundaries of significance when testing, I selected 50 as the default minimum number of iterations with additional iterations added based on the number of groups. More groups have more possible solutions, and therefore two solutions may be more likely to be close to each other. When using OGRS, I also recommend

making a visualization of the effect of the relationship between X and Y across M, such that you might be able to notice if a two transition points have been skipped, as this would likely show up on a graph as a quick change in the group differences.

There were no situations where a solution was found that was not a Johnson-Neyman transition point. Because of the way the algorithm is written there is no reason to suspect this might occur. The major issue of inaccuracy that could occur is for two transition points to be skipped.

Accuracy of Solutions

Though this algorithm has been developed to provide solutions to the Johnson-Neyman boundaries of significance for cases where there is no previously available solution, it is important to test this algorithm in situations where the solutions are known. Testing against known solutions allows us to understand better how accurate the solutions are in known cases, and how accurate the solutions are even in the unknown cases. Based on our previous discussion we know that closed form solutions are available in the two and three condition case.

Table 5.1 provides the closed form solution for two randomly generated datasets. The first dataset had two groups, and was generated to have two Johnson-Neyman solutions. The second dataset had three groups, and was generated to have two Johnson-Neyman solutions. The "true" solutions were calculated using Maple, an analytical software with highly accurate algorithms for computing the roots of polynomials. OGRS was used in SPSS and SAS with convergence criteria ranging from 10^{-2} to 10^{-8}. In the cells of Table 5.1 are two values, the first value is the solution from OGRS printed to eight decimal places, the second value is the proportion of the range of the moderator by which the OGRS solution differs from the Maple solution.

For example, if the moderator ranged from -1 to 1, then the range would be 2. If the Maple solution was 1.4 and the OGRS solution was 1.35. Then the difference between the Maple solution and the OGRS solution would be $1.4 - 1.35 = .05$ which is $.05/2 = .025$ or 2.5% of the range of the moderator. I used this statistic to describe the error in the OGRS algorithm, as it is reasonable to suspect the amount of error in the algorithm will be directly proportional to the scale of the moderator.

Based on Table 5.1 it seems that the algorithm is fairly accurate with the maximum error being less than .5% of the range of the moderator. It does seem that the SAS algorithm is slightly more accurate than the SPSS algorithm, particularly at higher values of the convergence criteria. This is likely related to the different criterion statistics used, where SAS uses an F-statistic and SPSS uses an α value. Additionally, as expected the algorithm is more accurate when a lower convergence criteria is used.

It seems that the algorithm performs admirably, even with very high convergence criteria. For researchers who believe that this level of accuracy is sufficient, they could use OGRS to solve for Johnson-Neyman boundaries of significance using this tool.

Though we cannot test for the situations of four groups or more, it seems reasonable to assume that the algorithm would remain accurate when there are more groups. The major issue of additional groups is missing potential transition points, and increased run time.

5.5.2 Run Time

Run times were recorded for both the SPSS and SAS version of OGRS using 27 different datasets. The datasets were randomly generated in GAUSS to be of varying group number, sample size, and number of Johnson-Neyman solutions. Three

Table 5.1: Accuracy of OGRS Johnson-Neyman Solutions for SPSS and SAS

		SPSS Solutions				SAS Solutions			
		Convergence Criteria				Convergence Criteria			
No. Groups	Maple Solutions	10^{-8}	10^{-6}	10^{-4}	10^{-2}	10^{-8}	10^{-6}	10^{-4}	10^{-2}
2	-0.5953813605	-0.59538141	-0.59538141	-0.59557986	-0.60329744	-0.59538137	-0.59538141	-0.59538141	-0.59557986
		1.30896E-08	1.30896E-08	5.24907E-05	0.002093306	2.51215E-09	1.30896E-08	1.30896E-08	5.24907E-05
	0.4201202797	0.42012028	0.42012018	0.42005403	0.42314106	0.42012028	0.42012028	0.42012018	0.42005403
		7.93312E-11	2.63644E-08	1.75189E-05	0.000798807	7.93312E-11	7.93312E-11	2.63644E-08	1.75189E-05
3	-0.393254492	-0.39325448	-0.39325335	-0.393478	-0.40605835	-0.39325451	-0.39325448	-0.39325335	-0.393478
		3.02555E-09	2.97756E-07	5.82963E-05	0.003339548	4.79915E-09	3.02555E-09	2.97756E-07	5.82963E-05
	1.032329674	1.03232966	1.03233182	1.03169531	1.03169531	1.03232967	1.03232966	1.03233182	1.03169531
		3.65153E-09	5.59727E-07	0.000165457	0.000165457	1.04329E-09	3.65153E-09	5.59727E-07	0.000165457

Solutions from SPSS and SAS using a variety of convergence criteria. The first value in each cell is the solution from OGRS to eight decimal places. The second value is the proportion of the range of the moderator by which the solution from OGRS differs from the Maple Solution.

different group numbers were used: 3, 5, and 7. Three sample sizes were used: 20, 100, and 1000. Finally there were three different numbers of solutions generated: 0, 1, and 2. Each dataset was analyzed 22 times in each language using each of two convergence criteria (10^{-8} and 10^{-4}) and each of 11 different initial iterations: 3, 4, 5, 10, 20, 30, 40, 50, 100, 1000, 10000.

Run times were recorded by hand on a cellphone stopwatch while running SPSS as there is not built in functional for timing in SPSS. The SAS program times each executed statement, so the times were recorded after each run. Both programs were run on a Dell Optiplex 745 computer running Windows 7 Enterprise with an Intel(R) Core(TM) 2 CPU processor. During the tests no other programs or applications were running.

Tables 5.2 to 5.4 contain all times for all conditions for both SPSS and SAS. Because times of less than 1 second by hand were fairly unreliable in SPSS, the time reported states "< 1". Minimum run time was .22 seconds (3 Groups, Sample Size 100, convergence criteria 10^{-4}, SAS, 3 initial iterations). Maximum run time was 1104 seconds or 18.39 minutes (5 Groups, Sample Size 1000, Convergence criteria 10^{-8}, SAS, 10000 initial iterations).

The major finding from this experiment was that smaller numbers of initial iterations took less time to run. However, smaller numbers of initial iterations are more likely to miss potential solutions. Balancing these two findings was important when setting the defaults for number of initial iterations. In general, a smaller number of initial iterations resulted in fewer probed points required to find a point which converged, leading to the quicker speed of these runs.

Datasets with more groups and larger samples took longer to analyze. This is likely because the data matrix needed to be inverted is larger in both of these cases. Inverting large matrices is time consuming even for computers. Especially when the number of initial iterations was quite large, some of the larger datasets took multiple minutes to run. Interestingly, though SAS was typically faster than SPSS, in some of these large data and large initial iteration conditions, SPSS ran faster than SAS.

The number of solutions influenced run time slightly. When there are no solutions, the program only checks the initial iterations then exits. In this situation the program runs quite quickly, especially when there are few initial iterations. When there is one solution, fewer magnifications must occur compared to when there are two solutions, so typically more solutions resulted in higher run times.

Convergence criteria seemed to impact runtime slightly. With a higher convergence criteria, fewer magnifications are required, and the program can run more quickly. Though the program run more quickly with a higher convergence criteria, lowering the convergence criteria impacts the accuracy of the solutions, and so the default was set to the most accurate, as run times were still fairly reasonable under this condition. If a researcher is particularly concerned about runtime and is willing to sacrifice some accuracy, they can change the convergence criteria from the default.

Some of these run times are fairly unreasonable and might lead a user to assume that their computer has crashed or the program is not working. As such, I chose the initial value of iterations to reflect fairly reasonable run times. Even in the worst case scenario tested (N = 1000, 7 Groups, and 2 Solutions) the default initial iterations would be 120, which we would expect to run in about 6 - 12 seconds depending on the language and convergence criteria. Additionally, by allowing the user to specify

the number of iterations, they can tailor the run time in cases where they have very large datasets or feel the program is likely to run slowly.

Overall, the runtimes informed the default convergence criteria and number of iterations. Keeping in mind a balance between accuracy and run time, I believe that aiming for the program to run in a few seconds in most small data situations and in under a minute in large data cases seemed reasonable. Researchers concerned that the program will run too long can change the defaults to their situation. The measured run times for this study were all on the same machine and are by no means universal. Some machines will run slower or faster. Users of OGRS can tailor their command line inputs to balance accuracy and speed on their own machine.

Next we will examine an application of OGRS to a real dataset. Through this example, users can see how what the OGRS output will look like and also how to interpret the output with respect to their research questions.

Table 5.2: Run Time in Seconds for OGRS with Three Groups

				Iterations										
Sample Size	No. Solutions	Conv. Crit.	Language	10000	1000	100	50	40	30	20	10	5	4	3
20	0	0.00000001	SPSS	1.43	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	0.75	0.36	0.41	0.25	0.25	0.33	0.27	0.30	0.36	0.34	0.30
		0.0001	SPSS	1.60	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	0.83	0.33	0.33	0.25	0.34	0.36	0.25	0.38	0.55	0.25	0.30
	1	0.00000001	SPSS	3.18	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	1.76	0.50	0.28	0.31	0.30	0.31	0.31	0.34	0.33	0.47	0.58
		0.0001	SPSS	2.46	1.05	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	1.31	0.48	0.31	0.36	0.36	0.31	0.31	0.25	0.30	0.28	0.31
	2	0.00000001	SPSS	4.95	1.20	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	2.64	0.67	0.38	0.47	0.36	0.48	0.42	0.31	0.31	0.33	0.36
		0.0001	SPSS	1.50	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	1.78	0.50	0.33	0.33	0.28	0.34	0.47	0.31	0.25	0.45	0.33
100	0	0.00000001	SPSS	2.68	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	2.33	0.45	0.34	0.31	0.31	0.31	0.42	0.38	0.30	0.53	0.33
		0.0001	SPSS	2.80	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	2.31	0.50	0.44	0.37	0.34	0.31	0.53	0.36	0.30	0.41	0.22
	1	0.00000001	SPSS	5.09	1.33	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	6.32	0.91	0.48	0.41	0.39	0.23	0.30	0.39	0.28	0.33	0.44
		0.0001	SPSS	2.64	1.03	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	4.37	0.80	0.42	0.34	0.41	0.42	0.36	0.44	0.34	0.36	0.30
	2	0.00000001	SPSS	12.03	1.80	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	10.44	1.53	0.44	0.50	0.37	0.41	0.30	0.48	0.39	0.36	0.41
		0.0001	SPSS	2.75	1.33	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	6.36	1.01	0.58	0.31	0.48	0.26	0.33	0.36	0.28	0.34	0.30
1000	0	0.00000001	SPSS	145.66	15.73	2.31	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	191.85	19.70	2.26	1.39	1.34	0.98	0.67	0.58	0.66	0.36	0.45
		0.0001	SPSS	146.57	15.51	2.33	1.49	1.36	1.21	< 1	< 1	< 1	< 1	< 1
			SAS	195.89	21.51	2.32	1.45	1.15	0.95	0.67	0.62	0.59	0.53	0.41
	1	0.00000001	SPSS	439.12	44.38	8.40	4.33	4.21	3.33	2.68	1.89	1.51	1.23	1.25
			SAS	544.96	73.49	9.50	5.82	4.68	4.07	3.29	2.00	1.50	1.34	1.11
		0.0001	SPSS	292.42	30.55	5.21	3.03	2.76	2.08	1.46	1.38	< 1	< 1	< 1
			SAS	366.76	37.24	5.82	4.12	3.31	2.56	2.29	1.20	0.91	0.92	0.72
	2	0.00000001	SPSS	590.78	78.83	12.33	7.13	6.63	1.25*	4.38	1.43*	< 1*	< 1*	< 1*
			SAS	879.86	123.96	16.01	9.89	9.28	0.904*	5.51	0.58*	0.44*	0.47*	0.44*
		0.0001	SPSS	438.70	45.51	8.36	4.46	3.70	1.15*	2.85	1.09*	< 1*	< 1*	< 1*
			SAS	528.58	53.26	9.16	6.55	5.40	0.967*	3.56	0.59*	0.36*	0.48*	0.37*

Run time in seconds from SPSS and SAS. Asterisks* indicate runs which did not produce all Johnson-Neyman points.

Table 5.3: Run Time in Seconds for OGRS with Five Groups

				Iterations										
Sample Size	No. Solutions	Conv. Crit.	Language	10000	1000	100	50	40	30	20	10	5	4	3
20	0	0.00000001	SPSS	1.85	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	0.97	0.41	0.34	0.61	0.48	0.48	0.75	0.5	0.39	0.31	0.37
		0.0001	SPSS	1.71	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	0.91	0.64	0.37	0.53	0.34	0.36	0.53	0.39	0.42	0.31	0.42
	1	0.00000001	SPSS	3.18	1.18	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	1.95	0.59	0.36	0.47	0.56	0.31	0.39	0.36	0.33	0.59	0.36
		0.0001	SPSS	1.85	1.03	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	1.45	0.47	0.34	0.48	0.37	0.36	0.36	0.37	0.39	0.48	0.44
	2	0.00000001	SPSS	5.41	1.38	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	3.09	0.72	0.37	0.36	0.39	0.41	0.42	0.41	0.39	0.45	0.55
		0.0001	SPSS	1.91	1.04	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	1.47	0.5	0.38	0.53	0.3	0.42	0.5	0.36	0.39	0.39	0.36
100	0	0.00000001	SPSS	3.48	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	2.87	0.66	0.44	0.42	0.51	0.37	0.39	0.47	0.36	0.44	0.36
		0.0001	SPSS	3.18	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	2.86	0.8	0.39	0.45	0.59	0.62	1.08	0.58	1.31	0.66	2.08
	1	0.00000001	SPSS	6.43	1.8	1.1	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	7.55	1.43	0.42	0.44	0.59	0.41	0.41	0.47	0.39	0.39	0.39
		0.0001	SPSS	3.4	1.43	1.06	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	5.23	0.77	0.47	0.47	0.44	0.39	0.42	0.45	0.38	0.39	0.37
	2	0.00000001	SPSS	10.55	2.34	1.16	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	12.28	1.7	0.51	0.62	0.39	0.41	0.56	0.31	0.36	0.45	0.45
		0.0001	SPSS	3.54	1.76	1.03	< 1	< 1	< 1	< 1	< 1	< 1	< 1	< 1
			SAS	7.71	1.33	0.55	0.42	0.45	0.44	0.45	0.59	0.36	0.44	0.52
1000	0	0.00000001	SPSS	176.54	18.61	2.75	1.8	1.68	1.45	1.36	1.03	< 1	< 1	< 1
			SAS	207.72	21.36	2.51	1.37	1.2	0.94	0.87	0.58	0.48	0.55	0.48
		0.0001	SPSS	185.25	19.31	3.33	1.91	1.7	1.71	1.3	1.16	< 1	< 1	< 1
			SAS	208.57	21.62	2.54	1.44	1.2	1.01	0.81	0.55	0.45	0.64	0.48
	1	0.00000001	SPSS	374.94	54.7	8.2	5.43	5.15	3.61	3.46	2.46	1.95	1.78	1.66
			SAS	622.46	62.51	10.66	6.47	5.37	4.62	3.15	2.15	1.61	1.48	1.33
		0.0001	SPSS	179.4	36.59	4.73	3.83	3.2	2.68	2.16	1.75	1.5	1.54	1.36
			SAS	414.35	41.92	6.52	3.42	3.65	2.78	2.37	1.36	1.01	0.97	1
	2	0.00000001	SPSS	533.09	93.26	14.96	8.61	7.93	7.08	5.08	3.66	2.68	2.53	1.35*
			SAS	1103.5	154.77	20.05	11.2	10.03	8.03	6.27	4.29	2.65	2.29	0.53*
		0.0001	SPSS	114.47	56.01	6.88	6.73	4.86	3.44	3.51	2.53	2.13	2.4	1.3*
			SAS	661.84	66.58	11.39	7.93	6.4	4.81	3.65	2.25	1.51	1.44	0.67*

Run time in seconds from SPSS and SAS. Asterisks* indicate runs which did not produce all Johnson-Neyman points.

Table 5.4: Run Time in Seconds for OGRS with Seven Groups

				Iterations										
Sample Size	No. Solutions	Conv. Crit.	Language	10000	1000	100	50	40	30	20	10	5	4	3
20	0	0.00000001	SPSS	2.74	1.33	1.3	1.2	1.18	1.25	1.13	1.06	1.35	1.2	1.2
			SAS	0.92	0.53	0.39	0.42	0.48	0.36	0.3	0.52	0.34	0.44	0.59
		0.0001	SPSS	2.44	1.43	1.33	1.26	1.14	1.2	1.18	1	1	1	1
			SAS	0.94	0.59	0.34	0.31	0.52	0.37	0.39	0.42	0.34	0.31	0.47
	1	0.00000001	SPSS	4.04	1.56	1.3	1.36	1.25	1.36	1.21	1.25	1.23	1.3	1.36
			SAS	2.21	0.69	0.55	0.36	0.48	0.39	0.45	0.56	0.41	0.36	0.67
		0.0001	SPSS	2.51	1.51	1.38	1.25	1.26	1.31	1.28	1.26	1.21	1.23	1.26
			SAS	1.55	0.67	0.34	0.34	0.5	0.34	0.33	0.55	0.37	0.33	0.47
	2	0.00000001	SPSS	6.9	1.83	1.6	1.21	1.28	1.26	1.16	1.2	1.21	1.31	1.31*
			SAS	3.5	0.73	0.36	0.34	0.32	0.49	0.35	0.34	0.31	0.35	0.29*
		0.0001	SPSS	2.51	1.56	1.3	1.31	1.33	1.36	1.15	1.39	1.39	1.35	1.36*
			SAS	2.17	0.57	0.39	0.4	0.4	0.4	0.39	0.39	0.4	0.41	0.49*
100	0	0.00000001	SPSS	4.53	1.7	1.64	1.24	1.38	1.36	1.43	1.45	1.38	1.43	1.43
			SAS	3.05	0.64	0.39	0.36	0.5	0.35	0.48	0.34	0.34	0.44	0.36
		0.0001	SPSS	4.61	1.78	1.41	1.43	1.36	1.41	1.33	1.39	1.38	1.38	1.53
			SAS	3.1	0.66	0.44	0.6	0.43	0.34	0.35	0.37	0.32	0.35	0.33
	1	0.00000001	SPSS	8.23	2.56	1.76	1.65	1.45	1.7	1.5	1.38	1.46	1.56	1.58
			SAS	8.59	1.45	0.5	0.47	0.46	0.44	0.42	0.39	0.39	0.47	0.38
		0.0001	SPSS	4.7	3.25	1.63	1.66	1.31	1.3	1.35	1.56	1.25	1.55	1.28
			SAS	5.79	0.88	0.39	0.36	0.39	0.36	0.35	0.31	0.33	0.63	0.34
	2	0.00000001	SPSS	12.56	3.31	1.84	1.66	1.78	1.7	1.53	1.6	1.59	1.59	1.58*
			SAS	13.99	2.11	0.58	0.53	0.42	0.43	0.52	0.4	0.36	0.39	0.34*
		0.0001	SPSS	4.8	2.56	1.96	1.6	1.8	1.68	1.4	1.76	1.61	1.61	1.58*
			SAS	8.54	1.28	0.52	0.51	0.43	0.41	0.42	0.4	0.32	0.47	0.36*
1000	0	0.00000001	SPSS	243.83	26.73	4.2	3.08	2.86	2.66	1.56	1.46	1.45	1.86	1.68
			SAS	224.68	22.99	2.71	1.53	1.34	1.11	0.89	0.75	0.54	0.74	0.51
		0.0001	SPSS	242.52	26.48	4.35	3.16	3.06	2.55	2.4	2.15	1.93	2	1.56
			SAS	225.06	22.85	2.62	1.5	1.36	1.05	1	0.61	0.49	0.61	0.46
	1	0.00000001	SPSS	479.12	51.21	9.3	5.79	7.36	6.06	4.06	3.3	3.16	3.08	2.76
			SAS	673.81	67.74	11.57	7.02	5.67	4.98	3.54	2.23	1.63	1.5	1.36
		0.0001	SPSS	242.2	26.91	6.91	4.55	5.03	4.3	3.53	2.76	2.78	2.46	2.48
			SAS	449.32	45.41	7.1	4.82	3.06	3.16	2.19	1.49	1.09	0.96	0.92
	2	0.00000001	SPSS	700.96	116.56	18.01	12.2	9.63	2.66*	2.64*	2.33*	1.63*	2.3*	1.71*
			SAS	895.33	112.24	18.1	10.39	9.16	1.05*	0.83*	0.67*	0.5*	0.45*	0.5*
		0.0001	SPSS	229.76	48.31	8.9	5.55	4.93	2.78*	2.48*	1.75*	2.24*	2.18*	2.2*
			SAS	448.32	67.61	9.56	5.96	5.04	1.16*	0.89*	0.78*	0.55*	0.56*	0.73*

Run time in seconds from SPSS and SAS. Asterisks* indicate runs which did not produce all Johnson-Neyman points.

Chapter 6: Party Differences in Support of Government Action to Mitigate Climate Change

Climate change has been an increasing topic of discussion throughout science and government. Particularly within psychology, it is important to understand who believes in climate change and who denies it. Through this example, we can investigate how political party identification and age are related to support of government action to mitigate climate change. In this study 815 U.S. citizens completed a survey online related to their opinions about global warming in the U.S. These participants were recruited such that they were approximately representative of the U.S. population.

The outcome I will examine in this dataset is a composite measure of how much respondents support actions of the U.S. government in response to climate issues. Each of the five questions related to a policy which would help the U.S. act in an attempt to mitigate climate change. Responses were measured on a scale of 1 (Strongly Opposed) to 7 (Strongly Support). An example item is "How much do you support or oppose increasing government investment for developing alternative energy like biofuels, wind or solar by 25%?" The scores on the five questions were averaged to give an overall score of support for government actions.

Participants also reported some of their political information such as their party identification (Democrat, Republican, or Independent). Additionally, some demographic information was collected such as age and gender.

Previous studies found that Democrats typically believe more that climate change exists and more strongly support addressing climate change than Republicans (Hoffman, 2011; McCright & Dunlap, 2011; Nisbet & Myers, 2007; Schuldt, Roh, & Schwarz, 2015), with Independents falling somewhere between Republicans and Democrats depending on the outcome variable (Guber, 2013).

Some studies have found, however, that younger individuals are more concerned about climate change and project worse climate outcomes (Joslyn & LeClerc, 2016), and they are also less willing to take risks with energy usage (e.g., nuclear and coal; Greenberg & Truelove, 2011) as compared to older individuals. This leads to the research question: Does the effect of party identification on support for government action depend on age? Particularly, is there an age at which there is no significant difference in support of government actions by party? This first question can be answered by testing the interaction between party identification and age in predicting support for government action against climate change. The second question can be answered using the Johnson-Neyman technique for categorical independent variables. Statistical tests for both of these questions are provided in the output of a single run of OGRS. In these analyses I will treat party identification (Democrat, Republican, or Independent) as the independent variable, age as the moderator, and support for government action as the outcome variable.

If the data are stored in a dataset called `global` where party identification is stored as a categorical variable called `partyid` where Democrats are coded as 1,

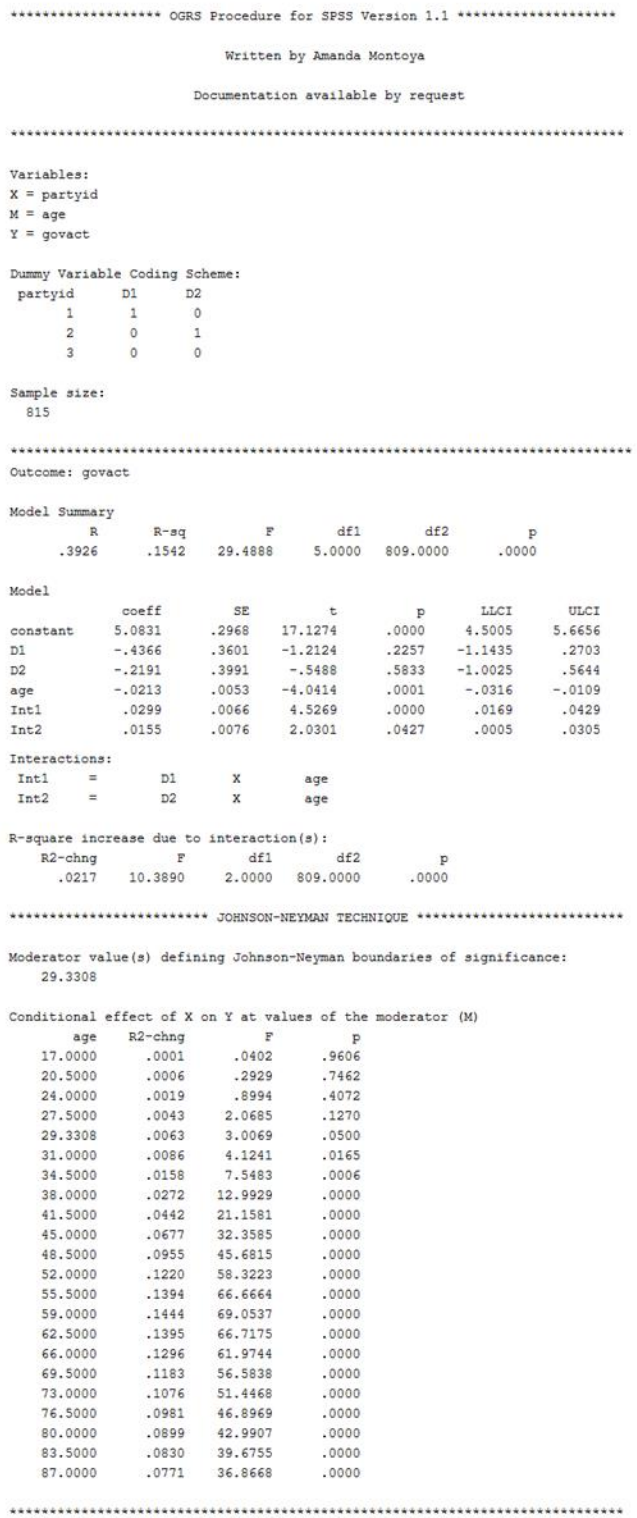
Independents are coded at 2, and Republicans are coded as 3, age (in years) of participant is stored as a continuous variable called `age`, and support for government action is stored as a continuous variable called `govact`, then the following OGRS commands analyze the data:

```
SPSS   OGRS  vars = partyid age govact /X = partyid /M = age
             /Y = govact.
SAS   %OGRS  (data = global, vars = partyid age govact,
              X = partyid, M = age, Y = govact);
```

Figure 6.1 and Figure 6.2 provide the output generated from running the above SPSS and SAS code respectively. In the first section of the output there is information about the variables, and how `partyid` has been recoded. Variable `D1` is an indicator for Democrats, variable `D2` is an indicator for Independents, and Republicans are the reference group.

The second section of the output is the regression model. Based on this output, it is clear that the regression coefficients for `D1` and `D2` are not significant ($b = -.44$, $p = .22$ and $b = -.2191$, $p = .5822$ respectively). This means that there are no significant differences in support for government action between Democrats and Republicans when age is zero, and there are no significant differences between Independents and Republicans when age is zero. This is not a particularly informative finding, as the minimum age in the data was 17, and reaching conclusions about newborns with party identifications based on this data is clearly overreaching. When `D1` and `D2` are both zero (which is the code for Republicans) age significantly predicts support in government actions ($b = -.0213$, $p = .0001$). This means that among Republicans, a year increase in age results in an expected .0213 unit decrease in support for government action against climate change. The coefficient for `D1M` ($b =$

Figure 6.1: OGRS Output for SPSS: Global Data



```
****************** OGRS Procedure for SPSS Version 1.1 ********************

                      Written by Amanda Montoya

                   Documentation available by request

***************************************************************************

Variables:
X = partyid
M = age
Y = govact

Dummy Variable Coding Scheme:
 partyid      D1      D2
       1       1       0
       2       0       1
       3       0       0

Sample size:
  815

***************************************************************************
Outcome: govact

Model Summary
          R       R-sq          F        df1        df2          p
      .3926      .1542    29.4888     5.0000   809.0000      .0000

Model
              coeff         SE          t          p       LLCI       ULCI
constant     5.0831      .2968    17.1274      .0000     4.5005     5.6656
D1           -.4366      .3601    -1.2124      .2257    -1.1435      .2703
D2           -.2191      .3991     -.5488      .5833    -1.0025      .5644
age          -.0213      .0053    -4.0414      .0001     -.0316     -.0109
Int1          .0299      .0066     4.5269      .0000      .0169      .0429
Int2          .0155      .0076     2.0301      .0427      .0005      .0305

Interactions:
 Int1     =        D1       X        age
 Int2     =        D2       X        age

R-square increase due to interaction(s):
    R2-chng          F        df1        df2          p
      .0217    10.3890     2.0000   809.0000      .0000

************************* JOHNSON-NEYMAN TECHNIQUE ****************************

Moderator value(s) defining Johnson-Neyman boundaries of significance:
    29.3308

Conditional effect of X on Y at values of the moderator (M)
        age    R2-chng          F          p
    17.0000      .0001      .0402      .9606
    20.5000      .0006      .2929      .7462
    24.0000      .0019      .8994      .4072
    27.5000      .0043     2.0685      .1270
    29.3308      .0063     3.0069      .0500
    31.0000      .0086     4.1241      .0165
    34.5000      .0158     7.5483      .0006
    38.0000      .0272    12.9929      .0000
    41.5000      .0442    21.1581      .0000
    45.0000      .0677    32.3585      .0000
    48.5000      .0955    45.6815      .0000
    52.0000      .1220    58.3223      .0000
    55.5000      .1394    66.6664      .0000
    59.0000      .1444    69.0537      .0000
    62.5000      .1395    66.7175      .0000
    66.0000      .1296    61.9744      .0000
    69.5000      .1183    56.5838      .0000
    73.0000      .1076    51.4468      .0000
    76.5000      .0981    46.8969      .0000
    80.0000      .0899    42.9907      .0000
    83.5000      .0830    39.6755      .0000
    87.0000      .0771    36.8668      .0000

***************************************************************************
```

Figure 6.2: OGRS Output for SAS: Global Data

********** OGRS Procedure for SAS Version 1.1 **********

Written by Amanda K. Montoya

Documentation available by request

****

Variables:

X =	PARTYID
M =	AGE
Y =	GOVACT

Dummy Variable Coding Scheme:

PARTYID	D1	D2
1	1	0
2	0	1
3	0	0

Sample Size:

815

****

Outcome:

GOVACT

Model Summary

R	R-Sq	F	df1	df2	p
0.3926	0.1542	29.4888	5.0000	809.0000	0.0000

Model

	coeff	SE	t	p	LLCI	ULCI
Constant	5.0831	0.2968	17.1274	0.0000	4.5005	5.6656
D1	-0.4366	0.3601	-1.2124	0.2257	-1.1435	0.2703
D2	-0.2191	0.3991	-0.5488	0.5833	-1.0025	0.5644
AGE	-0.0213	0.0053	-4.0414	0.0001	-0.0316	-0.0109
Int1	0.0299	0.0066	4.5269	0.0000	0.0169	0.0429
Int2	0.0155	0.0076	2.0301	0.0427	0.0005	0.0305

Interactions:

Int1	=	D1	X	AGE
Int2	=	D2	X	AGE

R-Square increase due to interaction(s):

R2-chng	F	df1	df2	p
0.0217	10.3890	2.0000	809.0000	0.0000

********** JOHNSON-NEYMAN TECHNIQUE **********

Moderator value(s) defining Johnson-Neyman boundaries of significance;

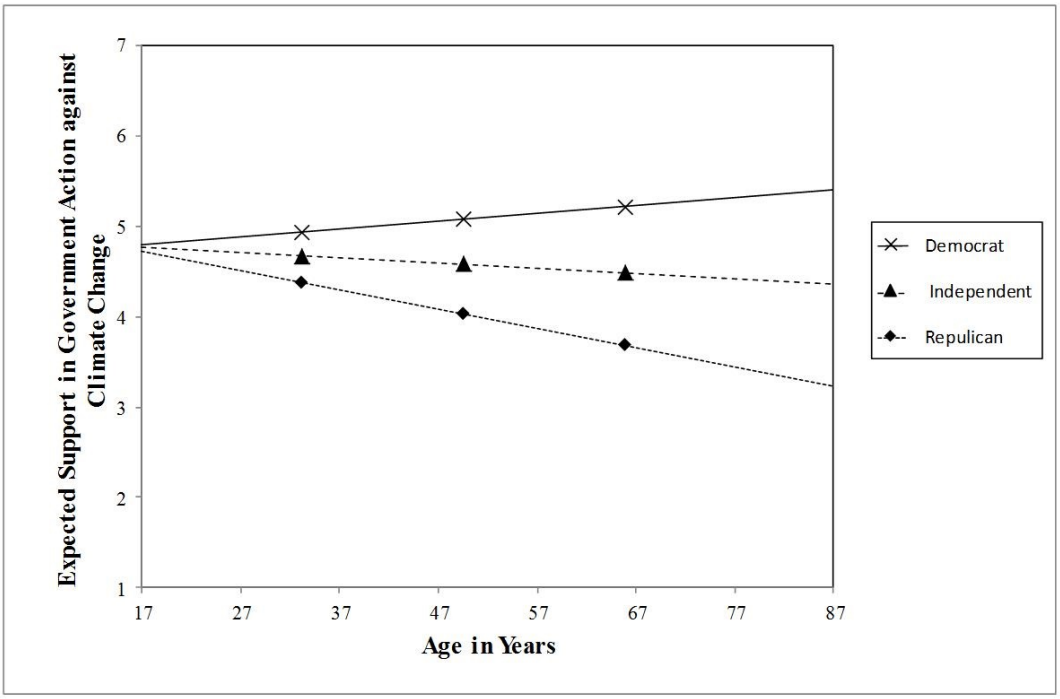
29.3308

Conditional effect of X on Y at values of the moderator (M)

AGE	R2-chng	F	p
17.0000	0.0001	0.0402	0.9606
21.3750	0.0008	0.4048	0.6673
25.7500	0.0029	1.3966	0.2480
29.3308	0.0063	3.0069	0.0500
30.1250	0.0073	3.5038	0.0305
34.5000	0.0158	7.5483	0.0006
38.8750	0.0309	14.7551	0.0000
43.2500	0.0552	26.3954	0.0000
47.6250	0.0884	42.2737	0.0000
52.0000	0.1220	58.3223	0.0000
56.3750	0.1418	67.8146	0.0000
60.7500	0.1429	68.3301	0.0000
65.1250	0.1323	63.2737	0.0000
69.5000	0.1183	56.5838	0.0000
73.8750	0.1051	50.2491	0.0000
78.2500	0.0938	44.8653	0.0000
82.6250	0.0846	40.4534	0.0000
87.0000	0.0771	36.8668	0.0000

****

Figure 6.3: Graph of Predicted Support for Government Action across Party Identification and Age



.03, $p < .001$), means that the difference between Republicans and Democrats in support for government action against climate change is expected to change as age changes. Specifically, as age increases by one year the difference between Democrats and Republicans is estimated to increase by .03 units, where democrats are estimated to be more supportive of government action relative to Republicans as age increases. This can be seen clearly in Figure 6.3, as age increases support for government action increases among Democrats, and decreases among Republicans.

Similarly, the coefficient for `D2M` ($b = .0155$, $p = .0427$), means that the difference between Republicans and Independents in support for government action against climate change is also estimated to increase as age increases. As age increases by one

year, the difference between Independents and Republicans is estimated to increase by .02 units, where independents are estimated to be more supportive of government action relative to Republicans as age increases. Based on Figure 2 it seems that Independents support decreases slightly as age increases, but the decrease for Republicans is stronger, and the gap between Independents and Republicans increases as age increases.

Based on the regression coefficients, it seems that there is a clear interaction between party identification and age in predicting support of government actions. However, there are many instances where one product coefficient is significant and the other is not, or both are marginally significant. In these cases, and in all cases, it is best to examine a formal test of whether allowing the relationship between the independent variable and the outcome to depend on the moderator increases the explained variance in the outcome variable. This test is provided at the bottom of the second section of the `OGRS` output labeled "R-square increase due to interaction(s)". This is the test comparing the model without either of the product coefficients, and thus fixing the effect of political identification on support of government action to be constant across age, to the model which allows this effect to vary across age. Based on the output allowing this relationship to vary across age explains an estimated additional 2% of the variance in support in government actions ($F(2, 809) = 10.39$, $p < .001$). This provides clear support for the hypothesis that the effect of political party on support for government action to mitigate climate change depends on age of respondents.

Based on this finding it would be reasonable to wonder "At what point does political party matter?" Alternatively, at what ages are there significant differences

between political parties on support for government action against climate change? This is a question which can be answered by using the Johnson-Neyman procedure for categorical independent variables. In the third section of the `OGRS` output labeled Johnson-Neyman Technique, there is output from the iterative program described in the previous chapter. The age at which the differences between political parties transitions from being significant to non-significant is 29.33 years. Based on the table below the Johnson-Neyman solutions, individuals below the age of 29.33 show no significant party divides on support for government action, but for individuals above the age of 29.33, party identification has a significant effect on support for government action against climate change. By defining regions of significance and regions of non-significance, the Johnson-Neyman procedure eliminates the need to probe at a variety of arbitrary points along the moderator. Instead based on these regions we know if the effect of party identification is significant for any point based on which of these regions it is contained in.

Chapter 7: Discussion

An approximate method for finding Johnson-Neyman boundaries of significance in an omnibus test of group difference should provide opportunities for researchers to answer new questions. Particularly in psychology, researchers often care about categorical variables such as ethnicity, and it is important that researchers have the ability to estimate moderation effects and probe them using the most sophisticated methods available. I began this thesis by explaining common methods of estimating and probing interaction effects in linear regression. I overviewed the history of the development of the Johnson-Neyman procedure, in order to show how the procedure has grown from use just in an ANCOVA to any linear regression. Using the principles of the Johnson-Neyman procedure in linear regression and the general linear model, I derived the boundaries of significance for a three category variable. Using the same method, I showed why the closed form derivation of the Johnson-Neyman boundaries of significance is not possible for more than four groups. I developed an iterative tool which can approximate these boundaries. I have shown that the tool achives very close solutions when the solutions are known and runs in a reasonable amount of time. I then showed how the tool is used and how the output is interpreted with a real data example about party differences in support for government action against global warming.

7.1 Uses and Non-Uses

My hope is that this tool will help researchers answer questions that they previously could not answer. There are many instances where other types of analyses are more appropriate for a question, and my aim is not for people to use this method in lieu of other more appropriate methods, but rather to use this method when it is most relevant to the question at hand. Many researchers with categorical variables are most interested in pairwise comparisons For example, when a researcher runs an experiment with two control conditions and an experimental condition, this may be an instance where the researcher is not particularly interested in omnibus group differences, and in this case the researcher could use the Johnson-Neyman procedures developed for pairwise comparisons. However, there are many instances where omnibus group differences are most relevant to a researcher's question of interest, and in that case I recommend using this tool over others where only the results for pairwise comparisons are available.

7.2 Future Directions

There are many extensions of this Johnson-Neyman approximation which may be of use to researchers. For example, allowing for multiple moderators may help researchers identify more complex regions of significance which are of higher dimension. With one moderator, there is only one dimension which needs to be searched, but with j moderators, there would be a j dimensional surface which would need to be searched for Johnson-Neyman transition points. In this situation the boundary of significance is a function, rather than a few points. This problem is complicated in two ways. First, developing a search algorithm to thoroughly search multidimensional

spaces is more difficult than the current one dimensional problem dealt with in this thesis. Second, there will be infinite transition points, and thus the goal might be to define a function which describes this transition point. The form of such a function is known in the continuous and dichotomous independent variable case (Abelson, 1953), but in the categorical independent variable case it is currently unknown.

Moderation can take many forms. Throughout this thesis I have dealt solely with linear moderation, where the effect of X on Y is a linear function of M. Other forms of moderation, like higher level polynomial functions, would require defining a different contrast matrix L. If the corresponding contrast matrix results in F being a polynomial function of M of any order, then the methods here could be applied to other types of moderation. However, not all contrast matrices will result in a polynomial function of M, and so the method proposed in this thesis is not generalizable to all types of moderation, but could potentially be generalized to other certain kinds of moderation.

Hayes and Matthes (2009) generalized the Johnson-Neyman procedure to linear regression with a dichotomous outcome. In principle, this could be done with the current method as well. Because logistic regression requires a maximum likelihood algorithm to estimate the coefficients and significance tests, and the Johnson-Neyman approximation proposed in this thesis requires iterative estimation of these models, program run time could become unreasonably long. A more efficient search algorithm could be used in combination with logistic regression to generalize this procedure to dichotomous outcomes.

Researchers are often interested in where along the range of a moderator the independent variable no longer has an effect (e.g., Voorhees, Baker, Bourdeau, Brocato,

& Cronin Jr., 2009), for whom some intervention would be effective (e.g., Oyserman, Brickman, & Rhodes, 2007), or who that may be particularly at risk for some outcome(e.g., Hilmer et al., 2008). One major misconception of the Johnson-Neyman procedure is that the boundaries of significance are good estimates of these points. Some researchers even make recommendations for treatments and decisions based on the findings of the Johnson-Neyman procedure. Other researchers have even compared transition points from one study to another, suggesting that the results are contradictory because the transition points were different (Carlson & Iacono, 2008). Because the boundaries of significance identify where an effect is statistically significant, they will be very dependent on study elements like sample size and effect size or strength of manipulation. Certain researchers may benefit from having an estimate of the point where an effect is zero, or the smallest it gets. These would not necessarily be the Johnson-Neyman boundaries of significance. The boundaries of significance will approach the point(s) along the moderator where the independent variable no longer has an effect as sample size increases. However, at any sample size, there is a better estimate of this point, which is the sample estimate of where the effect is zero. When there are two groups, a good estimate of when an effect is zero is the estimate of where the two groups have the same expected value on the outcome. For example, if X is a dichotomous variable coded 0 and 1, and the regression model is:

$$E(Y_i) = b_0 + b_1 X_i + b_2 M_i + b_3 X_i M_i.$$

Then the expected value of Y when X is 0 is

$$E(Y_i | X = 0) = b_0 + b_2 M_i,$$

and the expected value of Y when X is 1 is

$$E(Y_i|X = 0) = b_0 + b_1 + b_2M_i + b_3M_i.$$

By setting these two equations equal to each other, and solving for M, this gives an estimate of M such that the effect of X is zero.

$$b_0 + b_2M_i = b_0 + b_1 + b_2M_i + b_3M_i$$

$$0 = b_1 + b_3M_i$$

$$b_1/b_3 = M$$

This would be a better estimate of the point where the effect of X on Y is zero. The sampling distribution of this point likely has an unusual shape, as the distribution of the quotient of two normally distributed variables is not neccesarily normal. Methods such as bootstrapping could be used to provide a confidence interval for this point. Generalizing this method to categorical independent variables could be more complex, as there may be no point along the moderator where the effect of X is estimated to be zero. In this case, it could be worthwhile to estimate the point where the effect of the independent variable on the outcome is smallest instead. Some investigation into this estimate and elaboration on how the Johnson-Neyman points are sample-size dependent and are not a good estimate of when an independent variable has no effect could be useful in ensuring appropriate use of the Johnson-Neyman procedure.

7.3 Conclusion

The Johnson-Neyman procedure has been continuously generalized to more and more situations since it's development. I believe this is because it is a useful statistical tool that helps researchers better understand and visualize interactions, which

can often be very complicated. My original goal in this thesis was to provide an analytical solution to the Johnson-Neyman boundaries of significance with a categorical independent variable with any number of groups. Through my investigation of this topic I discovered that a true analytical solution would not be possible; however, I developed an iterative computer program which provides good approximations to the Johnson-Neyman boundaries under the conditions tested in this thesis, and I believe this tool could be helpful to researchers investigating questions of moderation of the effect of a categorical independent variable on some outcome.

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Appendix A: OGRS Macro for SPSS

The following is the macro code for OGRS in SPSS. This code, unaltered, typed into a syntax window and run in SPSS will define OGRS such that it can be used as described in this manuscript.

```
/*OGRS for SPSS Version 1.1*/.
/* Copyright 2016 */.
/* by Amanda Kay Montoya */.
/* Documentation available by email to montoya.29@osu.edu */.

preserve.
set printback=off.

define CDFINVT (p = !charend('/') /df = !charend('/')).
compute p0=-.322232431088.
compute p1 = -1.
compute p2 = -.342242088547.
compute p3 = -.0204231210245.
compute p4 = -.0000453642210148.
compute q0 = .0993484626060.
compute q1 = .588581570495.
compute q2 = .531103462366.
compute q3 = .103537752850.
compute q4 = .0038560700634.
compute ppv = !p.
do if (!p > .5).
compute ppv = 1-!p.
end if.
compute y5=sqrt(-2*ln(ppv)).
compute xp=y5+((((y5*p4+p3)*y5+p2)*y5+p1)*y5+p0)/((((y5*q4+q3)*y5+q2)*y5+q1)
*y5+q0).
do if (!p <= .5).
```

```
compute xp = -xp.
end if.
compute toutput = sqrt(!df*(exp((!df-(5/6))*(xp**2)/(!df-(2/3)+.1/!df)**2)
-1)).
!enddefine.

define CORR (var1 = !charend('/') /var2 = !charend('/')).
COMPUTE var1 = !var1.
COMPUTE var2 = !var2.
COMPUTE MeanV1 = csum(var1)/nrow(var1).
COMPUTE MeanV2 = csum(var2)/nrow(var2).
COMPUTE Var1Cent = var1 - MeanV1.
COMPUTE Var2Cent = var2 - MeanV2.
COMPUTE crosprod = csum(Var1Cent &* Var2Cent).
COMPUTE Var1SS = csum(Var1Cent &* Var1Cent).
COMPUTE Var2SS = csum(Var2Cent &* Var2Cent).
COMPUTE rPears = crosprod / (sqrt(var1SS)*sqrt(var2SS)).
!enddefine.

define RedR (center = !charend('/')).
COMPUTE mcent = m - !center.
COMPUTE m2int = MAKE(N, numgroup-1, -999).
LOOP i4 = 1 to (numgroup-1).
COMPUTE m2int(:,i4) = mcent&*data(:,i4+1).
END LOOP.
COMPUTE datam2 = {MAKE(N,1,1), mcent, m2int}.
DO IF covtog = 1.
COMPUTE datam2 = {datam2, data(:,(2*numgroup+1):ncol(data))}.
END IF.
COMPUTE yestm2 = datam2*inv(t(datam2)*datam2)*t(datam2)*y.
CORR var1 = y /var2 = yestm2.
COMPUTE  ycorm2 = rPears.
COMPUTE redr2 = ycorm2**2.
!enddefine.

define PROBE (min = !charend('/') /max = !charend('/')).
COMPUTE jump = (!max - !min)/iter.
COMPUTE dim = (transtog=0)*(iter+1) + (transtog=1)*(iter-1).
COMPUTE tempres = MAKE(dim, 8, -999).
DO IF transtog = 0.
COMPUTE i2 = !min.
ELSE IF transtog = 1.
COMPUTE i2 = !min+jump.
END IF.
LOOP i = 1 to dim.
```

```
RedR center = i2.
COMPUTE tempres(i, 1:2) = {i2, redr2}.
COMPUTE i2 = i2 + jump.
END LOOP.
COMPUTE tempres(:,3) = fullr2 - tempres(:,2).
COMPUTE tempres(:,4) = (dffull*tempres(:,3))&/(dfred*(1-fullr2)).
COMPUTE tempres(:,5) = 1-FCDF(tempres(:,4), dfred, dffull).
!enddefine.

define OGRS (vars = !charend('/') /x = !charend('/') /m = !charend('/')
/y = !charend('/') /conf = !charend('/') !default(95) /convcrit =
!charend('/') !default(.00000001) /decimals = !charend('/') !default(F10.4)
/iter = !charend('/') !default(0)).
set mxloop = 100000000.
matrix.
GET allvars /variables = !vars /names = allnames /missing = OMIT.
GET xdat /variables = !x /names = xname /missing = OMIT.
GET mdat /variables = !m /names = mname /missing = OMIT.
GET ydat /variables = !y /names = yname /missing = OMIT.
COMPUTE convcrit = !convcrit.
COMPUTE conf = !conf.
COMPUTE alpha = 1-(conf/100).
COMPUTE covtog = (ncol(allvars) - 3 > 0).
COMPUTE N = nrow(allvars).
DO IF covtog =1.
COMPUTE covcount = 1.
COMPUTE cov = MAKE(N, ncol(allvars) - 3, 999).
COMPUTE covname = MAKE(1, ncol(allvars)-3, 999).
END IF.
COMPUTE allvars = {xdat, allvars}.
COMPUTE allvars(GRADE(allvars(:,1)),:) = allvars.
COMPUTE allvars = allvars(:,2:ncol(allvars)).
LOOP i = 1 to ncol(allnames).
DO IF (allnames(:,i) = xname).
COMPUTE x = allvars(:,i).
ELSE IF (allnames(:,i) = mname).
COMPUTE m = allvars(:,i).
ELSE IF (allnames(:,i) = yname).
COMPUTE y = allvars(:,i).
ELSE.
DO IF covtog = 1.
COMPUTE cov(:,covcount) = allvars(:,i).
COMPUTE covname(:,covcount) = allnames(:,i).
COMPUTE covcount = covcount +1.
END IF.
```

```
END IF.
END LOOP.
COMPUTE designX = design(x).
COMPUTE numgroup = ncol(designX).
COMPUTE designX = designX(:,1:(numgroup-1)).
COMPUTE xmat = MAKE(ncol(designX)+1, ncol(designX)+1, -999).
LOOP kloop = 1 to ncol(designX).
LOOP i = 1 to N.
DO IF (designx(i,kloop) = 1).
COMPUTE xmat(kloop,1) = x(i,1).
END IF.
END LOOP IF xmat(kloop,1) <> -999.
END LOOP.
LOOP i = 1 to N.
DO IF all(designx(i,:)=0).
COMPUTE xmat(ncol(designX)+1,:) = {x(i,1), MAKE(1,ncol(designX), 0)}.
END IF.
END LOOP if xmat(ncol(designX)+1,1) <> -999).
COMPUTE xmat(1:(numgroup-1),2:numgroup) = Ident(numgroup-1).
COMPUTE prodcol = MAKE(N, ncol(designX), 999).
LOOP i = 1 to ncol(designX).
COMPUTE prodcol(:,i) = designX(:,i)&*m.
END LOOP.
DO IF covtog = 0.
COMPUTE data = {MAKE(N,1,1), designX, m, prodcol}.
ELSE IF covtog = 1.
COMPUTE data = {MAKE(N,1,1), designX, m, prodcol, cov}.
END IF.
DO IF (!iter = 0).
COMPUTE iter = 50+10*numgroup.
ELSE.
COMPUTE iter = !iter.
END IF.
COMPUTE yest = data*inv(t(data)*data)*t(data)*y.
CORR var1 = y /var2 = yest.
COMPUTE  ycor = rPears.
COMPUTE fullr2 = ycor**2.
COMPUTE dffull = N - ncol(data).
COMPUTE dfred = numgroup - 1.
COMPUTE Ffull = (fullr2*dffull)/((1-fullr2)*(ncol(data)-1)).
COMPUTE pfull = 1-FCDF(Ffull, (ncol(data)-1), dffull).
COMPUTE modres = MAKE(ncol(data), 6, -999).
COMPUTE modres(:,1) = inv(t(data)*data)*t(data)*y.
COMPUTE ssr = csum((y - yest)&**2).
COMPUTE msr = ssr/(N-ncol(data)).
```

```
COMPUTE semat = (msr*inv(t(data)*data)).
COMPUTE modres(:,2) = (diag(semat))&**(1/2).
COMPUTE modres(:,3) = modres(:,1)&/modres(:,2).
COMPUTE modres(:,4) = 2*(1-tcdf(abs(modres(:,3)),dffull)).
COMPUTE temp = alpha/2.
CDFINVT p = temp /df = dffull.
COMPUTE tcrit = toutput.
COMPUTE modres(:,5) = modres(:,1) - tcrit*modres(:,2).
COMPUTE modres(:,6) = modres(:,1) + tcrit*modres(:,2).
DO IF covtog = 0.
COMPUTE dataint = {MAKE(N,1,1), designX, m}.
ELSE IF covtog = 1.
COMPUTE dataint = {MAKE(N,1,1), designX, m, cov}.
END IF.
COMPUTE yestint = dataint*inv(t(dataint)*dataint)*t(dataint)*y.
CORR var1 = y /var2 = yestint.
COMPUTE  ycorint = rPears.
COMPUTE r2int = ycorint**2.
COMPUTE rchint = fullr2 - r2int.
COMPUTE Fint = (dffull*rchint)&/(dfred*(1-fullr2)).
COMPUTE pint = 1-FCDF(Fint, dfred, dffull).
COMPUTE intres = {rchint, Fint, dfred, dffull, pint}.
COMPUTE transtog = 0.
COMPUTE minM = cmin(m).
COMPUTE maxM = cmax(m).
PROBE min = minM /max = maxM.
COMPUTE results = tempres.
COMPUTE OGres = tempres.
COMPUTE results(nrow(results),6:7) = {0,0}.
COMPUTE i3 = 1.
LOOP IF i3 <= nrow(results).
DO IF i3 < nrow(results).
COMPUTE results(i3, 6) = 1*(results(i3,4) < results(i3+1,4)) -
1*(results(i3,4) > results(i3+1,4)).
COMPUTE results(i3, 7) = -1*((results(i3,5) < alpha) AND
(results(i3+1,5) > alpha)) + 1*((results(i3,5) > alpha) AND
(results(i3+1, 5) < alpha)).
END IF.
COMPUTE results(i3,8) = (abs(results(i3,5) - alpha) < convcrit).
DO IF i3 = nrow(results).
COMPUTE transcnv = 0.
ELSE IF i3 = 1.
COMPUTE transcnv = ((results(i3,7) = 1)AND((results(i3,8)=1)OR
(abs(results(i3+1,5) - alpha) < convcrit))).
ELSE.
```

```
COMPUTE trnscnv1 = ((results(i3,7) = 1) AND ((results(i3,8)=1) OR
 (abs(results(i3+1,5) - alpha) < convcrit))).
COMPUTE trnscnv2 = ((results(i3,7) = -1) AND ((results(i3,8)=1) OR
 (abs(results(i3+1,5) - alpha) < convcrit))).
COMPUTE transcnv = (trnscnv1 = 1) OR (trnscnv2 = 1).
END IF.
DO IF ((abs(results(i3,7))=1)AND(transcnv = 0)).
COMPUTE trnsindx = i3.
COMPUTE transtog = 1.
COMPUTE minmtran = mmin({results(i3+1,1), results(i3,1)}).
COMPUTE maxmtran = mmax({results(i3+1,1), results(i3,1)}).
PROBE min = minmtran /max = maxmtran.
COMPUTE results = {results; tempres}.
COMPUTE results(GRADE(results(:,1)),:) = results.
ELSE.
COMPUTE i3 = i3+1.
END IF.
END LOOP.
COMPUTE numJN = 1*(results(nrow(results),8) =1) + 1*((results(1,8) = 1) AND
 (results(1,7) <> 1)) + csum(abs(results(:,7))).
DO IF numJN > 0.
COMPUTE JNSoln = MAKE(numJN,1, -999).
COMPTUE JNIndx = MAKE(numJN, 1, -999).
COMPUTE slncnt = 1.
DO IF results(nrow(results),8) = 1.
COMPUTE JNSoln(1,1) = results(nrow(results),1).
COMPUTE JNIndx(1,1) = nrow(results).
COMPUTE slncnt = slncnt +1.
END IF.
LOOP i1 = 1 to nrow(results).
DO IF abs(results(i1,7)) = 1.
COMPUTE abvblw = {results(i1,1), abs(results(i1,5)-alpha); results(i1+1, 1),
 abs(results(i1+1,5) - alpha)}.
COMPUTE minval = GRADE(abvblw(:,2)).
COMPUTE indxtog = all(abvblw(GRADE(abvblw(:,2)),:) = abvblw).
DO IF (indxtog = 1).
COMPUTE JNIndx(slncnt,1) = i1.
ELSE.
COMPUTE JNIndx(slncnt,1) = i1+1.
END IF.
COMPUTE abvblw(GRADE(abvblw(:,2)),:) = abvblw.
COMPUTE JNSoln(slncnt,1) = abvblw(1,1).
COMPUTE slncnt = slncnt+1.
END IF.
END LOOP.
```

```
END IF.
PRINT /title = "****************** OGRS Procedure for SPSS Version 1.1
 ********************".
PRINT /title = "                              Written by Amanda Montoya       ".
PRINT /title = "                          Documentation available by request ".
PRINT /title = "************************************************************
******************".
COMPUTE varrlabs = {'X =', 'M = ', 'Y = '}.
PRINT {xname; mname; yname} /title = "Variables:" /rnames = varrlabs
/format = A8.
DO IF covtog = 1.
PRINT {covname} /title = "Statistical Controls:" /format = A8.
END IF.
COMPUTE dummylab = {"D1", "D2", "D3", "D4", "D5", "D6", "D7", "D8", "D9"}.
COMPUTE xmatlab = {xname, dummylab(1:(numgroup-1))}.
PRINT xmat /title = "Dummy Variable Coding Scheme:" /cnames = xmatlab.
PRINT N /title = "Sample size:".
PRINT {yname} /title = "*****************************************************
************************" /rlabels = "Outcome:" /format = A8.
COMPUTE modsum = {sqrt(fullr2), fullr2, Ffull, (ncol(data)-1), dffull, pfull}.
PRINT modsum /title = "Model Summary" /clabels = "R", "R-sq", "F" , "df1" ,
"df2", "p" /format = !decimals.
COMPUTE intlab = {"Int1", "Int2", "Int3", "Int4", "Int5", "Int6", "Int7",
"Int8", "Int9"}.
COMPUTE modlabs = {"constant", dummylab(1,1:(numgroup-1)), mname,
 intlab(1,1:(numgroup-1))}.
DO IF (covtog = 1).
COMPUTE modlabs = {modlabs, covname}.
END IF.
PRINT modres /title "Model" /rnames = modlabs /clabels = "coeff" , "SE", "t",
 "p", "LLCI", "ULCI" /format = !decimals.
COMPUTE intmat = MAKE((numgroup-1), 5, -999).
COMPUTE intmat(:,1) = t(intlab(1, 1:(numgroup-1))).
COMPUTE intmat(:,2) = MAKE((numgroup-1), 1, "=").
COMPUTE intmat(:,3) = t(dummylab(1, 1:(numgroup-1))).
COMPUTE intmat(:,4) = MAKE((numgroup-1), 1, "X").
COMPUTE intmat(:,5) = MAKE((numgroup-1), 1, mname).
PRINT intmat /title = "Interactions:" /format = A8.
PRINT intres /title = "R-square increase due to interaction(s):" /clabels =
"R2-chng" "F" "df1" "df2" "p" /format = !decimals.
PRINT /title = "************************ JOHNSON-NEYMAN TECHNIQUE
 *************************".
DO IF (iter > 10).
COMPUTE last = nrow(OGres).
COMPUTE rjump = rnd(last/20).
```

```
COMPTUE rowsel = 1.
COMPUTE rcount = 1+rjump.
LOOP IF (rcount <= last).
COMPUTE rowsel = {rowsel, rcount}.
COMPUTE rcount = rcount + rjump.
END LOOP.
DO IF (rcount - rjump <> last).
COMPUTE rowsel = {rowsel, last}.
END IF.
END IF.
COMPUTE JNtabnam = {mname, "R2-chng", "F", "p"}.
DO IF numJN > 0.
PRINT JNSoln /title = "Moderator value(s) defining Johnson-Neyman boundaries
of significance:" /format = !decimals.
DO IF (iter > 10).
COMPUTE JNouttab = {OGres(rowsel,:); results(JNIndx, :)}.
ELSE.
COMPUTE JNouttab = {OGres(:,:);results(JNIndx,:)}.
END IF.
COMPUTE JNouttab(GRADE(JNouttab(:,1)),:) = JNouttab.
COMPUTE JNouttab = JNouttab(:,{1,3,4,5}).
PRINT JNouttab /title = "Conditional effect of X on Y at values of the
moderator (M)" /cnames = JNtabnam /format = !decimals.
ELSE.
PRINT /title = "No Johnson-Neyman bounds found within range of observed
data".
DO IF (iter > 10).
COMPUTE JNouttab = OGres(rowsel,{1,3,4,5}).
ELSE.
COMPUTE JNouttab = OGres(:,{1,3,4,5}).
END IF.
PRINT JNouttab /title = "Conditional effect of X on Y at values of the
moderator (M)" /cnames = JNtabnam /format = !decimals.
END IF.
PRINT /title = "************************************************************
**********".
end matrix.
!enddefine.
restore.
```

Appendix B: OGRS Macro Documentation for SPSS

```
OGRS VARS = xvar mvar yvar [cov1 cov2 ...]
            /X = xvar /M = mvar /Y = yvar
            [/CONF = {c}{95**}]
            [/CONVCRIT = {cc}{.00000001**}]
            [/ITER = {it}{0**}]
            [/DECIMALS = {dec}{F10.4**}].
```

Subcommands in brackets are optional.

** Default if subcommand is omitted.

B.1 Overview

OGRS is a macro that estimates a linear regression model where the effect of the categorical independent variable (X) on the outcome (Y) is allowed to depend linearly on a moderator (M). OGRS provides all least squares regression estimates, with standard errors, t-statistics, p-values, and confidence intervals. As well, OGRS provides a test of interaction, using hierarchical regression analysis, comparing a model where the effect of X is allowed to vary linearly with M and a model where the effect of X is fixed across M.

OGRS also provides a unique method for probing the effect of X on Y using an approximation of the Johnson-Neyman procedure. OGRS searches the observed range of the moderator for points at which the effect of X on Y transitions from significant to non-significant or vice versa, as specified by some level of confidence (`CONF`). OGRS prints the transition points if any exist within the observed range of the moderator, as well as a table of points along the moderator, statistics related to the estimated effect of X at that point, and inferential statistics for the effect of X.

B.2 Preparation for Use

The OGRS.sps file should be opened as a syntax file in SPSS. Once opened, execute the entire file exactly as is. Do not modify the code. Once the program is executed, the OGRS.sps file window can be closed. Once executed, access to the OGRS command is available until quitting SPSS. The OGRS.sps file must be loaded and re-executed each time SPSS is opened.

B.3 Model Specification

Because OGRS will only accept one variable in the X subcommand, your independent variable should be coded into one variable, with unique codes for each group. Categorical independent variables can represent groups (e.g. race, political party), experimental conditions, or any other categorical variable of interest. Both the moderator M and the outcome variable Y are treated as continuous variables. Covariates specified in the model can be continuous or dichotomous, but they cannot be categorical with more than two groups. To use categorical covariates, using your desired coding scheme (e.g. dummy coding) to create $k - 1$ new variables to represent your

categorical covariate, and include all of these variables as covariates in the model. Covariates can be included in the `vars` subcommand, and they will be included in the regression model.

Various options in OGRS allow you to tailor the output to your question of interest. You can specify a confidence level, the convergence criteria for the Johnson-Neyman approximation, number of initial iterations in the Johnson-Neyman approximation, and the number of decimal places printed in the output. For example:

```
OGRS  vars = Xvar Mvar Yvar Cov1 /conf = 90 /convcrit = .0001
      /iter = 1000 /decimals = F10.6.
```

will estimate the effect of a categorical variable `Xvar` on `Yvar` moderated by `Mvar`. All confidence intervals will be 90% confidence intervals, and the Johnson-Neyman procedure will solve for points of transition along `Mvar` where the effect of `Xvar` on `Yvar` is exactly significant at $\alpha = 0.10$. The convergence criteria will be .0001 rather than the typical .00000001, and the Johnson-Neyman approximation will begin with 1000 iterations. All output will be printed to six decimals places.

B.4 Confidence Level

The `c` argument in the `CONF` subcommand specifies the confidence level for all confidence intervals and the criterion value for which the Johnson-Neyman procedure will compute the boundaries of significance. The default is 95%. Users can specify any confidence level greater than 50 and less than 100. For example `CONF = 90` will result in 90% confidence intervals and for the Johnson-Neyman procedure to find the points along the moderator at which the effect of the independent variable on the outcome variable is exactly significant at $\alpha = .10$.

B.5 Convergence Criteria

The `cc` argument in the `CONVCRIT` subcommand specifies the convergence criteria for the Johnson-Neyman algorithm in finding the boundaries of significance. The default is .00000001 (up to eight decimal places in both SPSS and SAS). Users can specify any number greater than .00000001. For example `CONVCRIT = .001` will mean that any solution which has a p-value within .001 of the specified α will be considered a sufficient solution for the Johnson-Neyman boundary of significance.

B.6 Initial Iterations

The `it` argument in the `ITER` subcommand specifies how many initial iterations should be used in the Johnson-Neyman algorithm. The default setting says 0 but this is used as an indicator that the user wants to use the default setting which is $50+10k$ where k is the number of groups in the variable specified in the X subcommand. Users can specify any whole number larger than 1 for this argument. For example, `ITER = 100` will result in the Johnson-Neyman algorithm dividing the range of the moderator into 100 sections in the initial search step. Users should be aware that large numbers of iterations may cause the program to run for a long time, so be patient. Additionally, very small numbers of iterations may cause the algorithm to miss potential transition points.

B.7 Decimals

The `dec` argument in the `DECIMALS` subcommand specifies how many decimal places are printed in the output. The default for this is F10.4. The user can specify

any format which is a valid printable numeric format (See SPSS Manual). For example `DECIMALS = F10.2` will print all outputs to two decimal places.

Appendix C: OGRS Macro for SAS

The following is the macro code for OGRS in SAS. This code, unaltered, typed into a syntax window and run in SAS will define OGRS such that it can be used as described in this manuscript.

```
*OGRS for SPSS Version 1.1;
* Copyright 2016;
* by Amanda Kay Montoya;
* Documentation available by email to montoya.29@osu.edu;

%macro RedR (center = );
mcent = m - &center;
m2int = mcent#data[,2:numgroup];
datam2 = J(Ninit,1,1)||mcent||m2int;
IF (covtog = 1) then datam2 = datam2||cov;
yestm2 = datam2*inv((datam2‘)*datam2)*datam2‘*y;
ycorm2 = CORR(yestm2||y);
ycorm2 = ycorm2[1,2];
redr2 = ycorm2**2;
%mend;

%macro PROBE (min = , max = );
jump = (&max - &min)/iter;
dim = (transtog = 0)*(iter+1) + (transtog = 1)*(iter-1);
tempres = J(dim, 8, -999);
IF (transtog = 0) then i2 = &min;
IF (transtog = 1) then i2 = &min + jump;
DO i = 1 TO dim;
%RedR(center = i2);
tempres[i, 1:2] = i2||redr2;
i2 = i2 + jump;
END;
tempres[,3] = fullr2 - tempres[,2];
```

```
tempres[,4] = (dffull*tempres[,3])/(dfred*(1-fullr2));
tempres[,5] = 1 - CDF('F',tempres[,4],dfred,dffull);
%mend;

%macro OGRS (data =, vars =, x = , m = , y = , conf = 95,
 convcrit = .00000001, decimals = 10.4, iter = 0);
proc iml;
start = time();
USE &data;
READ all var{&vars} into allvars;
allnames = {&vars};
READ all var{&x} into xdat;
xname = {&x};
READ all var{&m} into mdat;
mname = {&m};
READ all var{&y} into ydat;
yname = {&y};
convcrit = &convcrit;
conf = &conf;
alpha = 1-conf/100;
covtog = (ncol(allvars) - 3 > 0);
Ninit = nrow(allvars);
IF covtog=1 THEN DO;
covcount = 1;
cov = J(Ninit, ncol(allvars)-3, 999);
covname = J(1, ncol(allvars)-3, "AAAAAAAAAAAA");
END;
DO i = 1 TO ncol(allnames);
IF (allnames[,i] = xname) THEN DO;
x = allvars[,i];
END;
IF (allnames[,i] = mname) THEN DO;
m = allvars[,i];
END;
IF (allnames[,i] = yname) THEN DO;
y = allvars[,i];
END;
IF  all(allnames[,i] ^= xname||mname||yname) THEN DO;
IF covtog = 1 THEN DO;
cov[,covcount] = allvars[,i];
covname[,covcount] = allnames[,i];
covcount = covcount + 1;
END;
END;
END;
```

```
designx = design(x);
numgroup = ncol(designx);
designx = designx[,1:(numgroup-1)];
xmat = J(ncol(designx)+1, ncol(designx) + 1, -999);
DO kloop = 1 TO ncol(designx);
icount = 1;
DO WHILE (xmat[kloop,1] = -999);
IF designx[icount,kloop] = 1 THEN xmat[kloop,1] = x[icount,1];
icount = icount + 1;
END;
END;
icount = 1;
DO WHILE (xmat[ncol(designx)+1,1] = -999);
IF all(designx[icount,] = 0) THEN DO;
xmat[ncol(designx)+1,] = x[icount,1]||J(1,ncol(designX),0);
END;
icount = icount +1;
END;
xmat[1:(numgroup-1),2:numgroup] = I(numgroup-1);
prodcol = designX#m;
IF covtog = 0 THEN data = J(Ninit,1,1)||designX||m||prodcol;
IF covtog = 1 THEN data = J(Ninit,1,1)||designX||m||prodcol||cov;
yest = data*inv(data‘*data)*data‘*y;
ycor = corr(y||yest);
ycor = ycor[1,2];
fullr2 = ycor**2;
IF &iter = 0 THEN iter = 50+10*numgroup;
IF &iter ^= 0 THEN iter = &iter;
dffull = Ninit - ncol(data);
dfred = numgroup - 1;
Ffull = (fullr2*dffull)/((1-fullr2)*(ncol(data)-1));
pfull = 1 - CDF('F',Ffull,(ncol(data)-1),dffull);
critF = FINV(conf/100, dfred, dffull);
modres = J(ncol(data), 6, -999);
modres[,1] = inv(data‘*data)*data‘*y;
ssr = sum((y-yest)##2);
msr = ssr/(Ninit - ncol(data));
semat = msr*inv(data‘*data);
modres[,2] = (vecdiag(semat))##(1/2);
modres[,3] = modres[,1]/modres[,2];
modres[,4] = 2*(1-CDF('t', abs(modres[,3]),dffull));
tcrit = TINV(1-alpha/2, dffull);
modres[,5] = modres[,1] - tcrit*modres[,2];
modres[,6] = modres[,1] + tcrit*modres[,2];
dataint = J(Ninit, 1,1)||designx||m;
```

```
IF (covtog = 1) THEN dataint = dataint||cov;
yestint = dataint*inv(dataint‘*dataint)*dataint‘*y;
ycorint = CORR(yestint||y);
ycorint = ycorint[1,2];
r2int = ycorint##2;
rchint = fullr2 - r2int;
Fint = (dffull*rchint)/(dfred*(1-fullr2));
pint = 1 - CDF('F', Fint, dfred, dffull);
intres = rchint||Fint||dfred||dffull||pint;
transtog = 0;
minM = min(m);
maxM = max(m);
%PROBE (min = minM, max = maxM);
results = tempres;
OGres = tempres;
results[nrow(results),6:7] = {0 0};
i3 = 1;
DO WHILE (i3 <= nrow(results));
IF(i3 < nrow(results)) THEN DO;
results[i3,6] = (results[i3,4] < results[i3+1,4])-(results[i3,4] >
results[i3+1,4]);
results[i3,7] = -1*((results[i3,4] > critF) & (results[i3+1,4] <
critF)) + ((results[i3,4] < critF) & (results[i3+1,4] > critF));
END;
results[i3,8] = (abs(results[i3,4] - critF) < convcrit);
IF (i3 = nrow(results)) THEN DO;
transcnv = 0;
END;
IF (i3 = 1) THEN DO;
transcnv = ((results[i3,7] = 1)&((results[i3,8]=1)|(abs(results[i3+1,8] -
 critF) < convcrit)));
END;
IF ((i3 ^= nrow(results))&(i3 ^= 1))THEN DO;
trnscnv1 = ((results[i3,7] = 1) & ((results[i3,8] = 1) |
 (abs(results[i3+1,8] - critF) < convcrit)));
trnscnv2 = ((results[i3,7] = -1) & ((results[i3,8] = 1) |
 (abs(results[i3+1,8] - critF) < convcrit)));
transcnv = ((trnscnv1 = 1) | (trnscnv2 = 1));
END;
IF ((abs(results[i3,7]) = 1) & (transcnv = 0)) THEN DO;
trnsindx = i3;
transtog = 1;
minmtran = min(results[i3+1,1]||results[i3,1]);
maxmtran = max(results[i3+1,1]||results[i3,1]);
%PROBE (min = minmtran, max = maxmtran);
```

```
results = results//tempres;
CALL sort(results,1);
END;
IF ((abs(results[i3,7]) = 0) | (transcnv = 1)) THEN i3 = i3+1;
END;
numJN = (results[nrow(results),8]=1)+((results[1,8] = 1) &
 (results[1,7] ^= 1)) + sum(abs(results[,7]));
IF (numJN > 0) THEN DO;
JNSoln = J(numJN, 1, -999);
JNIndx = J(numJN, 1, -999);
slncnt = 1;
IF (results[nrow(results),8] = 1) THEN DO;
JNSoln[1,1] = results[nrow(results),1];
JNIndx[1,1] = nrow(results);
slncnt = slncnt + 1;
END;
DO i1 = 1 to nrow(results);
IF (abs(results[i1,7]) = 1) then do;
abvblw = (results[i1,1]||abs(results[i1,4] - critF))//(results[i1+1,1]||
abs(results[i1+1,4] - critF));
unsort = abvblw;
CALL sort(abvblw,2);
JNSoln[slncnt,1] = abvblw[1,1];
indxtog = all(abvblw = unsort);
IF (indxtog = 1) THEN JNIndx[slncnt,1] = i1;
IF (indxtog = 0) THEN JNIndx[slncnt,1] = i1+1;
slncnt = slncnt + 1;
END;
END;
END;
PRINT "********************** OGRS Procedure for SAS Version 1.1
 ************************";
PRINT "Written by Amanda K. Montoya";
PRINT "Documentation available by request";
PRINT "*****************************************************
***********************************";
varrlabs = {"X = " "M = " "Y = "};
PRINT (xname//mname//yname) [label = "Variables:" rowname = varrlabs];
IF (covtog = 1) THEN DO;
PRINT covname [label = "Statistical Controls:"];
END;
dummylab = {"D1" "D2" "D3" "D4" "D5" "D6" "D7" "D8" "D9"};
xmatlab = xname||dummylab[1,1:(numgroup-1)];
PRINT xmat [label = "Dummy Variable Coding Scheme:" colname = xmatlab];
PRINT Ninit [label = "Sample Size:"];
```

```
PRINT "************************************************************************
************************";
PRINT yname [label = "Outcome:"];
modsum = sqrt(fullr2)||fullr2||Ffull||(ncol(data)-1)||dffull||pfull;
PRINT modsum [label = "Model Summary" colname = {"R" "R-Sq" "F" "df1" "df2"
"p"} format = &decimals];
intlab = {"Int1" "Int2" "Int3" "Int4" "Int5" "Int6" "Int7" "Int8" "Int9"};
modlabs = "Constant"||dummylab[1,1:(numgroup-1)]||mname||intlab[1,1:
(numgroup-1)];
IF (covtog = 1) THEN modlabs = modlabs||covname;
PRINT modres [label = "Model" rowname = modlabs colname = {"coeff" "SE" "t"
"p" "LLCI" "ULCI"} format = &decimals];
intmat = J((numgroup-1),5,"AAAAAAAAAAAA");
intmat[,1] = (intlab[1,1:(numgroup-1)])`;
intmat[,2] = J((numgroup-1),1, "=");
intmat[,3] = (dummylab[1,1:(numgroup-1)])`;
intmat[,4] = J((numgroup-1),1,"X");
intmat[,5] = J((numgroup-1), 1, mname);
PRINT intmat [label = "Interactions:"];
PRINT intres [label = "R-Square increase due to interaction(s):"
colname = {"R2-chng" "F" "df1" "df2" "p"} format = &decimals];
PRINT "************************ JOHNSON-NEYMAN TECHNIQUE
************************";
IF (iter > 10) THEN DO;
last = nrow(OGres);
rjump = ceil(last/20);
rowsel = 1;
rcount = 1+rjump;
DO WHILE (rcount <= last);
rowsel = rowsel||rcount;
rcount = rcount +rjump;
END;
IF (rcount-rjump ^= last) THEN rowsel = rowsel||last;
END;
JNtabnam = mname||"R2-chng"||"F"||"p";
IF (numJN > 0) THEN DO;
PRINT JNSoln [label = "Moderator value(s) defining Johnson-Neyman boundaries
 of significance;" format = &decimals];
IF (iter > 10) THEN DO;
JNouttab = OGres[rowsel,]//results[JNIndx,];
END;
IF (iter <= 10) THEN DO;
JNouttab = OGres//results[JNIndx,];
END;
CALL sort(JNouttab,1);
```

```
JNouttab = JNouttab[,{1 3 4 5}];
END;
IF (numJN = 0) THEN DO;
PRINT "No Johnson-Neyman bounds found within the range of observed data";
IF (iter > 10) THEN JNouttab = OGres[rowsel,{1 3 4 5}];
IF (iter <= 10) THEN JNouttab = OGres[,{1 3 4 5}];
END;
PRINT JNouttab [label = "Conditional effect of X on Y at values of the
moderator (M)" colname = JNtabnam format = &decimals];
PRINT "**********************************************************
******************";
quit;
%mend;
```

Appendix D: OGRS Macro Documentation for SAS

```
OGRS (DATA = filename, VARS = xvar mvar yvar [cov1 cov2 ...],
                                    X = xvar, M = mvar, Y = yvar
                                              [,CONF = {c}{95**}]
                                  [,CONVCRIT = {cc}{.00000001**}]
                                              [,ITER = {it}{0**}]
                                   [,DECIMALS = {dec}{10.4**}].
```

Subcommands in brackets are optional.

** Default if subcommand is omitted.

D.1 Overview

OGRS is a macro that estimates a linear regression model where the effect of the categorical independent variable (X) on the outcome (Y) is allowed to depend linearly on a moderator (M). OGRS provides all least squares regression estimates, with standard errors, t-statistics, p-values, and confidence intervals. As well, OGRS provides a test of interaction, using hierarchical regression analysis, comparing a

model where the effect of X is allowed to vary linearly with M and a model where the effect of X is fixed across M.

OGRS also provides a unique method for probing the effect of X on Y using an approximation of the Johnson-Neyman procedure. OGRS searches the observed range of the moderator for points at which the effect of X on Y transitions from significant to non-significant or vice versa, as specified by some level of confidence (`CONF`). OGRS prints the transition points if any exist within the observed range of the moderator, as well as a table of points along the moderator, statistics related to the estimated effect of X at that point, and inferential statistics for the effect of X.

D.2 Preparation for Use

The OGRS.sas file should be opened as a program file in SAS. Once opened, execute the entire file exactly as is. Do not modify the code. Once the program is executed, the OGRS.sas file window can be closed. Once executed, access to the OGRS command is available until quitting SAS. The OGRS.sas file must be loaded and re-executed each time SAS is opened.

D.3 Model Specification

Because OGRS will only accept one variable in the X subcommand, your independent variable should be coded into one variable, with unique codes for each group. Categorical independent variables can represent groups (e.g. race, political party), experimental conditions, or any other categorical variable of interest. Both the moderator M and the outcome variable Y are treated as continuous variables. Covariates

specified in the model can be continuous or dichotomous, but they cannot be categorical with more than two groups. To use categorical covariates, using your desired coding scheme (e.g. dummy coding) to create $k-1$ new variables to represent your categorical covariate, and include all of these variables as covariates in the model. Covariates can be included in the `vars` subcommand, and they will be included in the regression model.

Various options in OGRS allow you to tailor the output to your question of interest. You can specify a confidence level, the convergence criteria for the Johnson-Neyman approximation, number of initial iterations in the Johnson-Neyman approximation, and the number of decimal places printed in the output. For example:

```
OGRS  (data = datname, vars = Xvar Mvar Yvar Cov1, conf = 90,
      convcrit = .0001, iter = 1000, decimals = 10.6.)
```

will estimate the effect of a categorical variable `Xvar` on `Yvar` moderated by `Mvar`. All confidence intervals will be 90% confidence intervals, and the Johnson-Neyman procedure will solve for points of transition along `Mvar` where the effect of `Xvar` on `Yvar` is exactly significant at $\alpha = 0.10$. The convergence criteria will be .0001 rather than the typical .00000001, and the Johnson-Neyman approximation will begin with 1000 iterations. All output will be printed to six decimals places.

D.4 Confidence Level

The `c` argument in the `CONF` subcommand specifies the confidence level for all confidence intervals and the criterion value for which the Johnson-Neyman procedure will compute the boundaries of significance. The default is 95%. Users can specify any confidence level greater than 50 and less than 100. For example `CONF = 90` will result in 90% confidence intervals and for the Johnson-Neyman procedure to find the

points along the moderator at which the effect of the independent variable on the outcome variable is exactly significant at $\alpha = .10$.

D.5 Convergence Criteria

The `cc` argument in the `CONVCRIT` subcommand specifies the convergence criteria for the Johnson-Neyman algorithm in finding the boundaries of significance. The default is .00000001 (up to eight decimal places in both SPSS and SAS). Users can specify any number greater than .00000001. For example `CONVCRIT = .001` will mean that any solution which has a an F-statistic within .001 of the criterion F-statistic will be considered a sufficient solution for the Johnson-Neyman boundary of significance.

D.6 Initial Iterations

The `it` argument in the `ITER` subcommand specifies how many initial iterations should be used in the Johnson-Neyman algorithm. The default setting says 0 but this is used as an indicator that the user wants to use the default setting which is $50+10k$ where k is the number of groups in the variable specified in the X subcommand. Users can specify any whole number larger than 1 for this argument. For example, `ITER = 100` will result in the Johnson-Neyman algorithm dividing the range of the moderator into 100 sections in the initial search step. Users should be aware that large numbers of iterations may cause the program to run for a long time, so be patient. Additionally, very small numbers of iterations may cause the algorithm to miss potential transition points.

D.7 Decimals

The `dec` argument in the `DECIMALS` subcommand specifies how many decimal places are printed in the output. The default for this is F10.4. The user can specify any format which is a valid printable numeric format (See SAS Manual). For example `DECIMALS = 10.2` will print all outputs to two decimal places.