Small Sample Mediation Analysis: How Far Can We Push the Bootstrap?

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Abstract

In statistical mediation analysis, the widely-used bootstrap confidence interval for the indirect effect may be more vulnerable to the influence of outliers in small samples than alternative popular tests due to its reliance on resampling of the data with replacement. In a Monte Carlo study, we find the opposite to be true. The percentile bootstrap confidence interval is actually *least* susceptible to the influence of outliers in small samples compared to other popularly-used tests.

Simple Mediation

- Mediation analysis is used to answer questions of "How" or "By what process" does *X* affect *Y*?
- The *indirect effect* of *X* on *Y* quantifies the sequence of causal steps by which *X* affects *Y* through a mediator variable *M*.
- Using OLS regression, the indirect effect of *X* can be estimated using two linear models:

(1) $M = i_M + aX + e_M$

(2) $Y = i_Y + bM + c'X + e_Y$

- The indirect effect of *X* on *Y* is quantified as the product of the *a* and *b* paths from equations 1 and 2 (and seen in Figure 1).
- It estimates how much two cases that differ by one unit on *X* are expected to differ on *Y* as a result of *X*'s influence on *Y* through *M*.
- The *c'* path in Figure 1 is called the *direct effect*. It quantifies the effect of *X* on *Y* independent of its influence through *M*.

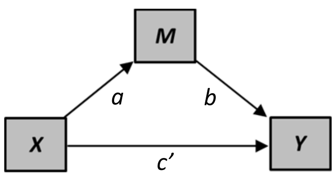

Figure 1. A diagram of simple mediation

The Monte Carlo Simulation

- By way of a Monte Carlo simulation, we generated samples from a population mediation process. We varied:
  - *a* path (0, .59), *b* path (0, .59), *c'* path (0, .35)
  - sample size (*n*=20, 40, 60)
  - number of outliers (0, 1, 2, 3)
  - Location of outliers (on *M*, on *Y*, on *M* and *Y*)
  - Method of introducing the outliers (Random *X*, Most extreme *X* in absolute value)
- *n* observations of *X* were drawn from a standard normal distribution. Values of *M* were then generated from equation (1), where e_M is a standard normal deviate in a no outliers condition. In an outlier condition, one, two or three e_M were from a uniform distribution ranging from 3 to 5, with sign randomized.
- The observations to which these uniform errors were added were either chosen randomly (i.e. assigned to a random *X*) or to the values of *M* paired with the most extreme absolute values of *X*.
- These values of *X* and *M* were then used to generate *Y* following equation (2) in the same way, with occasional outliers or not.
- The indirect effect was tested using the test of joint-significance, the Sobel test, the Monte Carlo CI (5000 samples), the percentile bootstrap CI, and the bias-corrected bootstrap CI (based on 5000 resamples). A .05 criterion for rejection was used (i.e., 95% CI).
- Type I error was tallied across 5000 replications per condition.

Results and Discussion

- Only Type I error rates are discussed here, tabulated below after collapsing across *a*, *b*, and *c'* conditions. Bradley (1978) suggested that the most liberal criterion of acceptance for Type I error was $.5\alpha \leq p \leq 1.5\alpha$, so those error rates larger than .075 are considered unacceptable under threat of outliers.
- When outliers are present only on *Y*, the tests we examined kept Type I error rate in control but were more conservative with outliers. Those results are absent from the table below.
- Overall, the percentile bootstrap CI performed best. Its false rejection rate remained fairly stable across conditions. Its success may be attributable to the resampling that was originally thought to be a problem. Just as it can resample the outliers repeatedly in a single bootstrap sample, it may not resample them at all. And bootstrap CIs don't require estimates of standard errors which may be heavily influenced by outliers.
- The tests most compromised by the introduction of outliers are the joint significance test and the Monte Carlo CI, most likely due to their reliance on estimates of standard errors that can be influenced by the presence of outliers.
- The Sobel test is very conservative in ideal circumstances, so its resistance to outliers in several conditions containing them can perhaps be attributed to this conservativism.
- Overall, outliers had the most destructive influence when they were located on both *M* and *Y*, followed by on *M* when the outliers were attached to the most extreme absolute value of *X*. But the percentile bootstrap CI still performed well even in these conditions.
- **Recommendation:** The percentile bootstrap can be pushed further than other methods. If unusual cases worry you in your small samples, use the percentile bootstrap CI for inference about the indirect effect, as its performance is relatively invulnerable to outliers in the conditions we simulated.

Mediation and Small Samples

- Large samples are generally better than small ones. Inferential tests have higher power in bigger samples but are also less vulnerable to the occasional "outlier" in the data.
- In a small sample, "outliers" can make up a larger proportion of the observations than in larger samples, allowing for greater influence on parameter estimates and standard errors.
- Popular inferential techniques for indirect effects such as the Sobel test, the joint-significance test, and the Monte Carlo confidence interval rely on estimates of the regression coefficients *a* and *b* and their standard errors. Inferences could be based off of four "poisoned" estimates influenced by unusual cases.
- Resampling methods such as the bootstrap confidence interval (CI) don't rely on standard errors but may be vulnerable to outliers because they resample the data with replacement, which may allow outliers to appear multiple times in a resample. We address this question in a Monte Carlo study.

Type I error rate			Joint Significance			Sobel Test			Monte Carlo CI			Percentile Bootstrap			BC Bootstrap CI		
		Outliers	*n*=20	*n*=40	*n*=60	*n*=20	*n*=40	*n*=60	*n*=20	*n*=40	*n*=60	*n*=20	*n*=40	*n*=60	*n*=20	*n*=40	*n*=60
On *M*		None	.021	.032	.034	.008	.015	.020	.025	.034	.036	.027	.038	.040	.049	.060	.057
	Random *X*	1	.021	.031	.033	.009	.015	.019	.025	.033	.035	.028	.038	.040	.042	.058	.057
		2	.020	.029	.033	.010	.015	.019	.026	.031	.035	.031	.038	.039	.047	.057	.057
		3	.021	.028	.032	.013	.015	.018	.026	.030	.032	.032	.038	.040	.048	.058	.059
	Most Extreme *X*	1	.077	.089	.078	.043	.057	.055	.091	.094	.081	.026	.037	.038	.059	.068	.065
		2	**.110**	**.117**	**.110**	.080	.091	.089	**.122**	**.122**	**.113**	.032	.040	.043	.068	.070	.071
		3	.090	**.103**	**.105**	.075	.085	.087	.098	**.107**	**.107**	.041	.043	.042	.075	.069	.067
On *M* and *Y*	Random *X*	1	.091	**.143**	**.138**	.046	.079	.091	**.103**	**.144**	**.141**	.023	.034	.041	.063	.087	.083
		2	.064	**.135**	**.155**	.042	.092	**.118**	.073	**.136**	**.155**	.019	.034	.041	.044	.088	.096
		3	.038	.092	**.123**	.027	.063	.090	.045	.093	**.124**	.019	.036	.047	.041	.078	.092
	Most Extreme *X*	1	**.136**	**.164**	**.172**	.086	.095	**.106**	**.158**	**.167**	**.172**	.022	.033	.041	.047	.092	**.110**
		2	**.141**	**.211**	**.222**	.097	**.145**	**.159**	**.157**	**.215**	**.222**	.021	.032	.039	.046	.083	**.108**
		3	.097	**.158**	**.195**	.074	**.117**	**.144**	**.109**	**.162**	**.196**	.025	.031	.042	.054	.076	.099

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